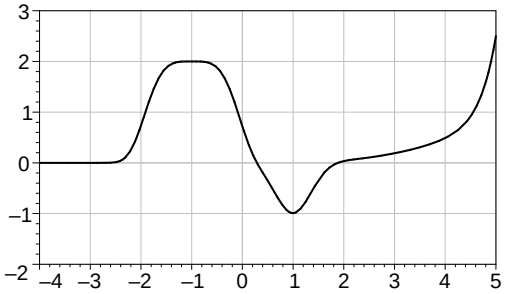
Partial Review for Midterm Exam MAT 125

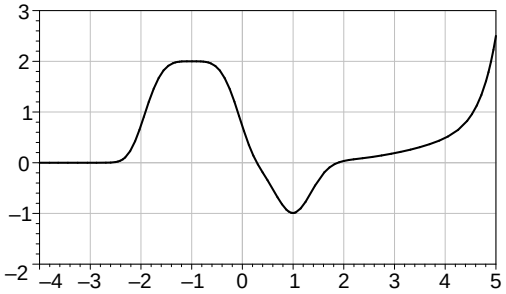
Here are some problems you can use to help prepare yourself for the exam, which cover derivative-related questions not on the other practice problems. Note that this is not an exhaustive set of problems: just because something is here doesn't mean it will be on the exam, and there may be material on the exam not represented here. You should not need a calculator to do any of these problems.

The exam will be held on Wednesday, October 15, at 8:30 PM. Do not forget to bring your student ID card or another photo ID like a driver's license.

1. At right is a graph of a function $f(x)$.
Draw a graph of the derivative $f'(x)$.
At which x values is f increasing?
At which x values is f concave up?



2. At right is a graph of the **derivative** $g'(x)$ of a function.
Draw the graph of a function g which has the given graph as its derivative.
At which x values is g increasing?
At which x values is g concave up?

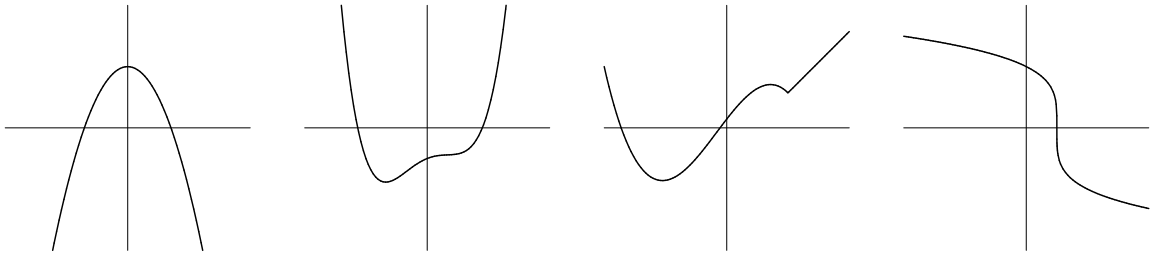


3. Let $f(x) = x^3 - 3x^2 + 2$.

a. Compute $f'(x)$ and find the formula of the tangent line to the graph of $f(x)$ through the point $(1, 0)$.

b. Compute $f''(2)$. Is $f(x)$ concave up or concave down at $x = 2$? Justify your answer.

4. The graphs of several functions $f(x)$ are shown below. On the same set of axes, sketch the function $f'(x)$.



5. Which of the following represents $f'(2)$ where $f(x) = e^{x^2}$.

$$\lim_{x\to 2}\frac{e^{x^2}-e^{a^2}}{h} \qquad \lim_{h\to 0}\frac{e^4(e^{4h+h^2}-1)}{h} \qquad \lim_{x\to 2}\frac{e^{x^2}-e^2}{x-2} \qquad \lim_{h\to 0}\frac{e^{(x^2+h^2)}-e^{x^2}}{h}$$

6. In the paragraph below is a description of how the amount of water $W(t)$ in a tub varied with time.

> The tub held about 50 gallons of green, brackish water, with some stuff floating in it that I didn't even want to guess about. I had to get it out of there. When I opened the drain the water drained out rapidly at first, but then it went slower and slower, until it stopped completely after about 5 minutes. The tub was about 1/4-full of that nasty stuff. Would I have to stick my hand in it? *Ick*— there was no way I could do that. I just stared at it for a couple of minutes, but then I got an idea. I dumped in about 10 gallons of boiling water. That did something: there was this tremendous noise like *BLUUUUURP*, and then the tub drained steadily, emptying completely in just a minute or so.

Use this description to sketch a graph of $W(t)$ and its derivative $W'(t)$. Pay careful attention to slope and concavity. Label the axes, with units.