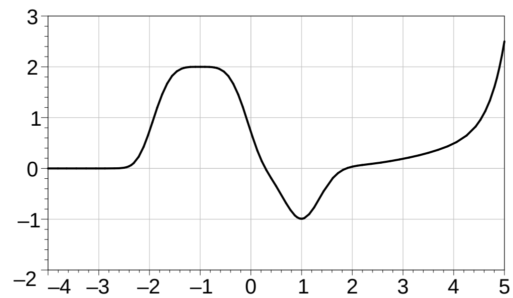
Review for Second Midterm Exam MAT 125

Here are some problems you can use to prepare yourself for the second exam. Note that this is not an exhaustive set of problems: just because something is here doesn't mean it will be on the exam, and there may be material on the exam not represented here. If you would like to use this review sheet as a sample exam, allow yourself about 2 hours.

The exam will be held on Tuesday, November 9, at 8:30 PM. Do not forget to bring your student ID card or another photo ID like a driver's license.

1. At right is a graph of a function $f(x)$.
Draw a graph of the derivative $f'(x)$.
At which x values is f increasing?
At which x values is f concave up?



2. For each of the following functions $f(x)$, compute the derivative $f'(x)$.

a. $f(x) = (x^3 - x)\ln(x)$

b. $f(x) = e^{2x} - 2x^e$

c. $f(x) = \left(\dfrac{x^2}{\tan(2x)}\right)^3$

d. $f(x) = \arctan\left(\sqrt{x}\right)$

e. $f(x) = e^{(x^2 - \sqrt{x})}$

f. $f(x) = \sqrt{\dfrac{5}{x} - \dfrac{x}{5}}$

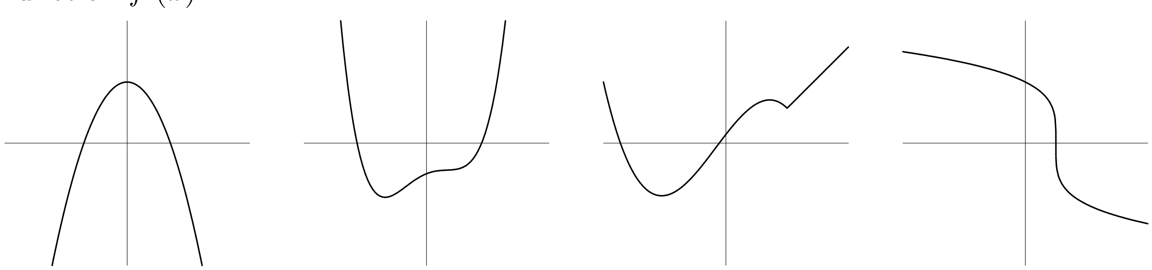
g. $f(x) = \cos(x)\sin(x)$

3. Let $f(x) = \ln(x)\cos(\pi x)$.

a. Compute $f'(x)$ and find the formula of the tangent line to the graph of $f(x)$ through the point $(1, 0)$.

b. Compute $f''(2)$. Is $f(x)$ concave up or concave down at $x = 2$? Justify your answer.

4. The graphs of several functions $f(x)$ are shown below. On the same set of axes, sketch the function $f'(x)$.



5. Let $f(x) = 2x^2 + xe^x$.

a. Compute $f'(x)$ and $f''(x)$.

b. For which of the following values of x, is $f(x)$ increasing near x? -1, 0, 2.

c. Is $f(x)$ concave up near $x = 0$?

d. Are there any values of x for which $f(x)$ is concave down near x?

(**Hint:** Remember that $e^t > 0$ for all values of t).

6. Do the following limits exist? If they do, compute them:

a. $\displaystyle\lim_{x\to\infty} \frac{7x^4+4x}{5x^5-6x+1}$

b. $\displaystyle\lim_{x\to-\infty} \frac{|x|}{x}$

c. $\displaystyle\lim_{x\to 2} \frac{1}{2-x}$

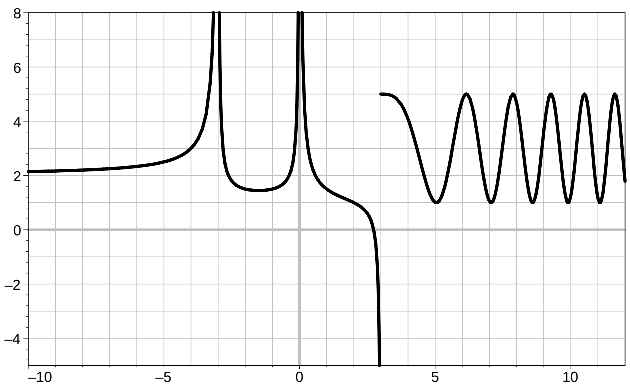
d. $\displaystyle\lim_{x\to 2} \frac{1}{2-x}$

e. $\displaystyle\lim_{x\to\infty} \frac{\sin(x^2)}{x}$

(**Hint:** Remember that $-1 \le \sin(t) \le 1$ for all values of t).

f. $\displaystyle\lim_{x\to 3^+} \ln(x^2-4x+3)$

7. At right is the graph of a function $f(x)$. Do the following limits exist? If they do, what are they?



a. $\displaystyle\lim_{x\to -3} f(x)$

b. $\displaystyle\lim_{x\to 0^+} f(x)$

c. $\displaystyle\lim_{x\to 3^-} f(x)$

d. $\displaystyle\lim_{x\to 3} f(x)$

e. $\displaystyle\lim_{x\to -\infty} f(x)$

f. $\displaystyle\lim_{x\to \infty} f(x)$

g. $\displaystyle\lim_{x\to \infty} \frac{f(x)}{x}$

8. Which of the following represents $f'(2)$ where $f(x) = e^{x^2}$.

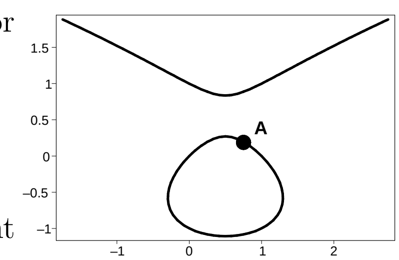
$$\lim_{x\to 2} \frac{e^{x^2}-e^{a^2}}{h} \qquad \lim_{h\to 0} \frac{e^4(e^{4h+h^2}-1)}{h} \qquad \lim_{x\to 2} \frac{e^{x^2}-e^2}{x-2} \qquad \lim_{h\to 0} \frac{e^{(x^2+h^2)}-e^{x^2}}{h}$$

9. If $h(0) = 1$, $h'(0) = 3$, $h(1) = 0$, $h'(1) = -1$, $h(2) = 0$, $h'(2) = 2$, and $f(x) = h(h(x))$, what is $f'(1)$?

10. Consider the curve C which consists of the set of points for which

$$x^2 - x = y^3 - y$$

(see the graph at right).



a. Write the equation of the line tangent to C at the point $(1, 0)$.

b. Use your answer to part **a** to estimate the y-coordinate of the point with x-coordinate $3/4$ marked A in the figure. Plug your estimate into the equation for C to determine how good it is.

11. In the paragraph below is a description of how the water level $W(t)$ in a tub varied with time.

> The tub held about 50 gallons of green, brackish water, with some stuff floating in it that I didn't even want to guess about. I had to get it out of there. When I opened the drain the water drained out rapidly at first, but then it went slower and slower, until it stopped completely after about 5 minutes. The tub was about 1/4-full of that nasty stuff. Would I have to stick my hand in it? *Ick*— there was no way I could do that. I just stared at it for a couple of minutes, but then I got an idea. I dumped in about 10 gallons of boiling water. That did something: there was this tremendous noise like *BLUUUUURP*, and then the tub drained steadily, emptying completely in just a minute or so.

Use this description to sketch a graph of $W(t)$ and its derivative $W'(t)$. Pay careful attention to slope and concavity. Label the axes, with units.