Final Exam MAT 125— Spring 1999

Name: **Section:**

Signature: **Mother's first name:**

Directions: There are 13 problems on 10 pages (including this cover sheet) in this exam. Make sure that you have them all. Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate what is where if you expect someone to look at it. Books and discussions with friends are not permitted. By signing above, you are affirming that you are the person whose name is on this page, and that all the work in this booklet is your own. Unless told otherwise, leave all your answers in exact form (that is, do *not* approximate π, square roots, and so on.)

problem	possible	score
1	8	
2	6	
3	8	
4	8	
5	8	
6	8	
7	12	
8	6	
9	12	

10	11	12	13	
8	8	8	8	

TOTAL	100	

1.*(8 points)* Answer the questions below by circling the best answer.

i. Evaluate $\lim_{x\to+\infty} \dfrac{x-8x^3}{4+x^3}$.

a) $+\infty$ **c)** $\dfrac{1}{4}$ **e)** -8

b) $\dfrac{1}{2}$ **d)** -2

ii. Let $x^4+xy^3-y^2=4$. Find $\dfrac{dy}{dx}$.

a) $\dfrac{x-2y^2}{3x+2}$ **c)** $\dfrac{2y-4x^3}{3xy^2}$ **e)** $\dfrac{y-3x}{4x^2-2}$

b) $\dfrac{4x^3+y^3}{2y-3xy^2}$ **d)** $\dfrac{4x^3-y}{xy^2+2}$

iii. Find an equation for the tangent line to the graph of $y=x^3+2x^2-5$ at the point $(1,-2)$.

a) $y+2=7(x-1)$ **c)** $y+2=4(x-1)$ **e)** $y-2=7(x+1)$

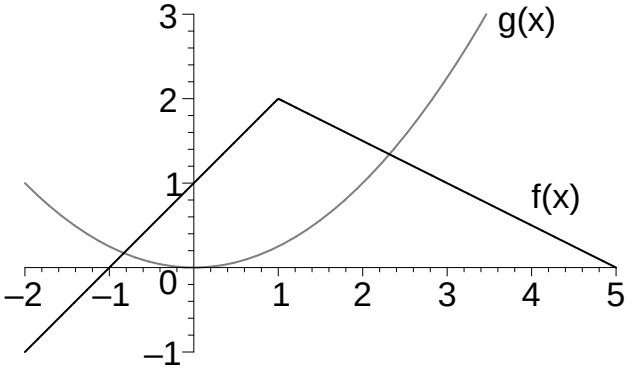
b) $y+2=x(x-1)$ **d)** $y-2=x-3$

iv. Let $f(x)=\sqrt{1-x^2}$. Find $f'(x)$.

a) $\dfrac{x^2}{\sqrt{1-x^2}}$ **c)** $\dfrac{-x}{\sqrt{1-x^2}}$ **e)** $\dfrac{-1}{2\sqrt{1-x^2}}$

b) $\dfrac{1}{2\sqrt{1-x^2}}$ **d)** $\sqrt{1-2x}$

2. *(6 points)* Given the graphs of f and g in the figure at right, find each of the following quantities.



a. $g(f(5))$

b. $g(g(0))$

c. $g(f(1))$

3. *(8 points)* Answer the questions below by circling the best answer.

i. The function given by $y = x^3 + 6x^2 - 63x + 10$ with $y' = 3(x-3)(x+7)$ is decreasing on

a) $(-7, 3)$ **c)** $(-\infty, -3)$ and $(7, +\infty)$ **e)** no interval

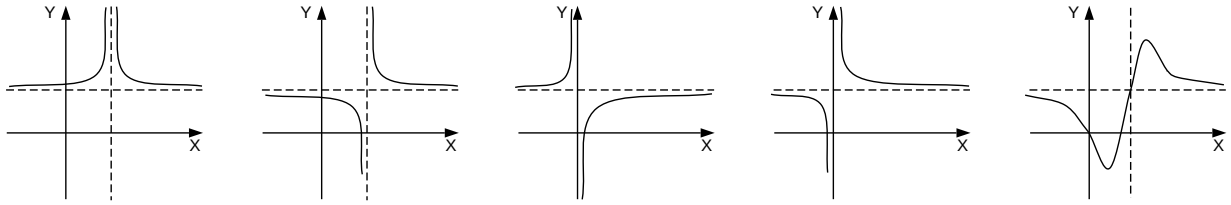
b) $(-\infty, -7)$ and $(3, +\infty)$ **d)** $(-3, -7)$

ii. Find the derivative of e^{2x}.

a) $2e^{2x}$ **c)** $e^x - x$ **e)** $2e^x$

b) $2xe^{x^2}$ **d)** $e^x \, ln\, x$

iii. Which of the following is the best sketch of the function $y = 3 + \dfrac{2}{x-3}$?

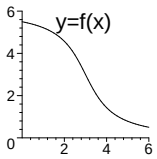


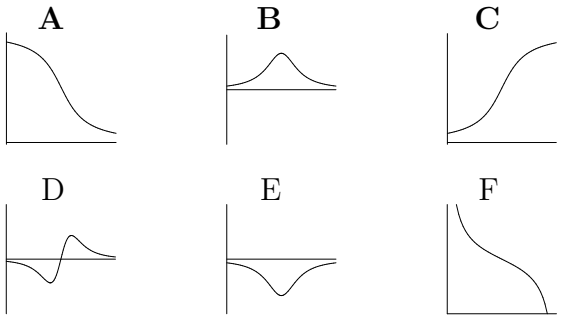
iv. Let $f(x) = \tan x = \dfrac{\sin x}{\cos x}$. Then $f'(x)$ is equal to

a) $\cot(x)$ **c)** $-\cot(x)$ **e)** $\dfrac{1}{1+x^2}$

b) $\dfrac{1}{\cos^2(x)}$ **d)** $\dfrac{-1}{1-x^2}$

4. *(8 points)* Let $f(x)$ be the function whose graph is shown at right. For each part (**i**) through (**iv**) below it, write the letter of the graph below which best represents the function described.





i. $f'(x)$

ii. $f^{-1}(x)$

iii. $6 - f(x)$

iv. $f''(x)$

5. *(8 points)* Answer the questions below by circling the best answer.

i. Suppose $f(x)$ has a derivative $f'(x)$ which is continuous for all x. Consider the table below.

x	$f'(x)$
1.0	2.37
1.1	0.31
1.2	−2.0
1.3	−3.5
1.4	−3.3
1.5	−1.7

x	$f'(x)$
1.6	0.76
1.7	2.80
1.8	3.61
1.9	2.76
2.0	0.69
2.1	−1.62

In which of the following intervals does $f(x)$ have a local minimum?

a) $(1.1, 1.4)$ **c)** $(1.8, 2.0)$ **e)** no interval

b) $(1.5, 1.7)$ **d)** $(2.0, 2.1)$

ii. A function $f(x)$ has derivative $f'(x) = 2\cos x + 4x - 2$. At the critical point $x = 0$, $f(x)$ has

a) a local maximum **c)** an undefined value **e)** none of these

b) a local minimum **d)** an inflection point

iii. The function $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2$ has a local minimum at

a) 0 **c)** $\sqrt{2}+1$ **e)** 4

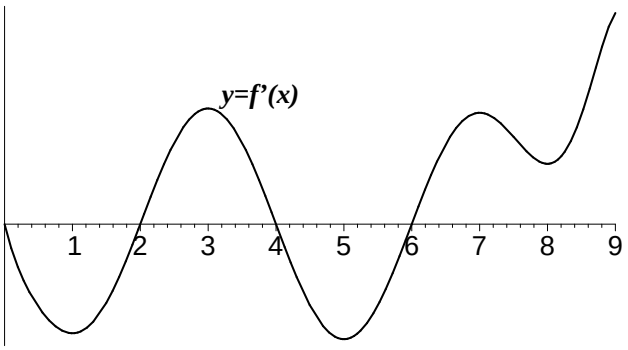
b) 1 **d)** $\sqrt{6}$

iv. Find the derivative of $\ln(3x)$.

a) $\frac{3}{x}$ **c)** $\frac{1}{3x}$ **e)** none of the above

b) $\frac{1}{x}$ **d)** $\frac{1}{x}+3$

6. *(8 points)* Let f be the function whose derivative is shown at right.



a. On what intervals is $f(x)$ increasing?

b. List all x values where f has a local maximum at x.

c. On what intervals is $f(x)$ concave up?

d. List all x values where f has an inflection point.

7. *(12 points)* Calculate each of the quantities below. If a limit does not exist, distinguish between $+\infty$, $-\infty$, and no limiting behavior (DNE). Give justification or show some work for each of your answers.

a. $\lim_{x\to 2} \dfrac{x^2-4}{x-2}$

b. $\lim_{x\to\infty} \cos\left(\dfrac{1}{x}\right)$

c. $\lim_{x\to 0} \dfrac{\ln(4+x)-\ln(4)}{x}$

d. $\lim_{x\to 0} x\left(\cos\left(\dfrac{1}{x^2}\right)\right)^2$

e. Calculate the derivative of $f(x) = x^3\cos(x)$.

f. Calculate the derivative of $f(x) = x\ln(2x^2+1)$

8. *(6 points)* For $-2 \le x \le 4$, at what x values do the absolute maximum and minimum values of $f(x) = x^3 - x^2 - 5x - 5$ occur? Justify your answer.

9. *(12 points)* Let $f(x) = e^{1-\frac{x^2}{2}}$.

a. Calculate $f'(x)$.

b. Calculate $f''(x)$.

c. For what values of x is $f(x)$ increasing?

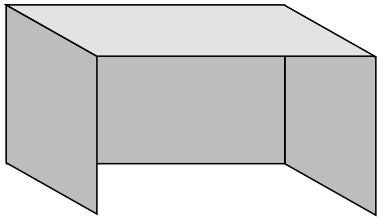
d. Does f have any local maxima? If so, at what x values do they occur?

e. Does f have any inflection points? If so, at what x values do they occur?

f. Calculate $\lim_{x\to\infty} f(x)$.

Do **any three** of the problems 10-13. Cross out the problem you do not want graded.

10.*(8 points)* Hiram wants to build a shed to park his tractor in and to keep the rain and snow off his bales of hay. The shed will have a roof, walls on three sides, an open front, and no floor. It must be at least 7' tall so that he doesn't bump his head, wide enough to get the tractor in and out easily (at least 8'), and exactly 10' deep. He plans to use redwood which costs \$1 per square foot for the two sides and the back, and corrugated tin (\$0.50 per square foot) for the roof. If he needs a volume of 2560 cubic feet to store everything, what dimensions will minimize the cost of the shed? Be sure to justify that your answer is indeed a minimum.



Do **any three** of the problems 10-13. Cross out the problem you do not want graded.

11.*(8 points)* Two aircraft are in the vicinity of the main air traffic control center in Neverland. Pachyderm flight 1047 from Celesteville to Paris is approaching from the south, and Peterpan flight 366 from London to Wendytown is approaching from the east. Both flights are at an altitude of 10,000 meters, on a path that takes them directly over the air traffic control center. The Pachyderm flight is approaching the center at a rate of 740 km/hr, while the Peterpan flight is approaching at a rate of 850 km/hr. At what rate is the distance between the two planes changing when the Pachyderm flight is 40km from Neverland and the Peterpan flight is 30 km from Neverland? Is the distance increasing or decreasing?

Do **any three** of the problems 10-13. Cross out the problem you do not want graded.

12. *(8 points)* When air expands without gaining or losing heat, its pressure P and volume V are related by the equation $V^{1.4} = 1/P$. Suppose that at a certain time, the volume is $243cm^3$ and the pressure is $80kPa$. If the pressure is decreasing at a rate of $10kPa/min$, at what rate is the volume increasing?

Do **any three** of the problems 10-13. Cross out the problem you do not want graded.

13. *(8 points)* A window is going to be made in the shape of a rectangle with an equilateral triangle on top of it. If the perimeter of the window is 30 ft, what dimensions maximize the amount of light let in? You may find it useful to know that the larger the area, the more light comes in, that the area of an equilateral triangle of side length s is $s^2\sqrt{3}/4$, and that light travels at about 186,000 miles per second.