Second Midterm Exam — Math 125, Fall 2004

Name: **ID#:** **Rec:**

ELC3	Josh	MW 6:50p	R01	Yoav	M 9:35a	R02	Samir	Th 12:50p	R03	Ari	Tu 2:20p	R04	Yuan	Th 5:20p
R05	Daniel	M 11:45a	R06	Samir	Th 2:20p	R07	Wenchuan	Th 5:20p	R08	Amy	F 9:35a	R09	Yuan	Th 2:20p
R10	Yoav	W 9:35a	R11	Daniel	W 11:45a	R12	Dezhen	F 11:45a	R13	Xiaojun	M 5:20p	R15	Amy	W 8:30a
R16	Ari	Th 2:20p	R17	Daniel	M 5:20p	R19	Dezhen	M 11:45a	R20	Wenchuan	Tu 9:50a			

problem	1	2	3	4	5	6	Total
possible	16	16	20	16	16	16	100
score							

Directions: There are 6 problems on six pages in this exam. Make sure that you have them all. Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate what is where if you expect someone to look at it. **Books, calculators, extra papers, and discussions with friends are not permitted.** Leave all answers in exact form (that is, do *not* approximate π, square roots, and so on.)

1. *(16 points)* Determine whether the following limits exist. If they do, find them. If the limit does not exist, distinguish between $+\infty$, $-\infty$, and no limiting behavior (DNE). Justify your answers.

a. $\displaystyle \lim_{x\to 3} \frac{x^2}{(x-3)^2} = +\infty$

b. $\displaystyle \lim_{x\to\infty} \frac{6x^5-7x^2+2}{5+x^3+2x^5} = \lim_{x\to\infty} \frac{\frac{1}{x^5}(6x^5-7x^2+2)}{\frac{1}{x^5}(5+x^3+2x^5)} = \lim_{x\to\infty} \frac{6-\frac{7}{x^3}+\frac{2}{x^5}}{\frac{5}{x^5}+\frac{1}{x^2}+2} = \frac{6-0+0}{0+0+2} = \boxed{3}$

c. $\displaystyle \lim_{h\to 0} \cos\left(\frac{\pi}{3}+h\right) = \cos\left(\frac{\pi}{3}+0\right) = \cos\frac{\pi}{3} = \boxed{\frac{1}{2}}$

Since $\cos x$ is a continuous function

d. $\displaystyle \lim_{h\to 0} \frac{\cos\left(\frac{\pi}{3}+h\right)-\frac{1}{2}}{h} = \lim_{h\to 0} \frac{\cos\left(\frac{\pi}{3}+h\right)-\cos\left(\frac{\pi}{3}\right)}{h} = \cos'\left(\frac{\pi}{3}\right) = -\sin\frac{\pi}{3} = \boxed{-\frac{\sqrt{3}}{2}}$

2.*(16 points)* A raindrop falls vertically from the clouds on a calm, drizzly day. During the time it takes for the raindrop to fall from the clouds to the ground, the altitude (vertical position) of the raindrop, measured in feet above sea level, is well approximated by

$$z(t) = 2000 - 12t - 4e^{-2t}$$

where t is measured in seconds.

a. Find the velocity $v(t)$ of the raindrop.

$$v(t) = z'(t) = (2000 - 12t - 4 \cdot e^{-2t})' = -12 - 4 \cdot (-2) \cdot e^{-2t} =$$

$$= \boxed{-12 + 8e^{-2t}}$$

$v(t) =$ ______________________________ ft/sec

b. Find the acceleration $a(t)$ of the raindrop.

$$a(t) = v'(t) = z''(t) = (-12 + 8e^{-2t})' = 8 \cdot (-2) \cdot e^{-2t} =$$

$$= \boxed{-16 \cdot e^{-2t}}$$

$a(t) =$ ______________________________ ft/sec^2

3. *(20 points)* Compute each of the derivatives:

a. $\dfrac{d}{dx}\left[\, x^7 - 5x^2 + 7 \,\right] = 7\cdot x^{7-1} - 5\cdot 2\cdot x^{2-1} + 0 =$

$= \boxed{7\cdot x^6 - 10\cdot x}$

b. $\dfrac{d}{d\theta}\left[\, \sin\left(\dfrac{\pi}{180}\theta\right) \,\right] = \dfrac{\pi}{180}\cdot \sin' x \Big|_{x=\frac{\pi}{180}\cdot\theta} = \boxed{\dfrac{\pi}{180}\cos\left(\dfrac{\pi}{180}\theta\right)}$

Chain Rule

c. $\dfrac{d}{dx}\left[\, 5(\arctan x)^2 \,\right] = 5\cdot 2\cdot(\arctan x)^{2-1}\cdot \dfrac{d}{dx}(\arctan x) =$

Chain Rule

$= \boxed{10\cdot \arctan x\cdot \dfrac{1}{1+x^2}}$

d. $\dfrac{d}{du}\left[\, \dfrac{\cos u}{u^2} \,\right] = \dfrac{\cos' u\cdot u^2 - (u^2)'\cdot\cos u}{(u^2)^2} = \dfrac{-\sin u\cdot u^2 - 2u\cdot\cos u}{u^4} =$

Quotient Rule

$= \dfrac{-u\,(u\cdot\sin u + 2\cos u)}{u^4} = \boxed{-\dfrac{u\sin u + 2\cos u}{u^3}}$

e. $\dfrac{d}{dx}\left[\, \ln(\sin x) \,\right] = \dfrac{d}{dx}(\sin x)\cdot\dfrac{1}{\sin x} = \dfrac{\cos x}{\sin x}.$

Chain Rule

4. *(16 points)* Let $f(x) = x\,e^{-4x}$. You may find it useful (or not) to recall that for any z, $e^z > 0$, or that we use the letter e in honor of Leonhard Euler (1707–1783).

a. Compute $f'(x)$.

$$(x\cdot e^{-4x})' \underset{\substack{\uparrow \\ \text{product rule}}}{=} (x)'\cdot e^{-4x} + x\cdot(e^{-4x})' = e^{-4x} + x\cdot(-4)e^{-4x} =$$

$$= \boxed{e^{-4x}(1-4x)}$$

b. Compute $f''(x)$.

$$(x\cdot e^{-4x})'' = ((x\cdot e^{-4x})')' = (e^{-4x}(1-4x))' = (e^{-4x})'\cdot(1-4x) +$$

$$+ e^{-4x}\cdot(1-4x)' = (1-4x)\cdot(-4)\cdot e^{-4x} + e^{-4x}\cdot(-4) =$$

$$= e^{-4x}\left((1-4x)\cdot(-4)+(-4)\right) = -4e^{-4x}(\cancel{1}-4x+\cancel{1}) =$$

$$= -4e^{-4x}\cdot(-4x) = \boxed{16xe^{4x}}$$

c. For what x values is $f(x)$ increasing?

$f'(x) = e^{-4x}(1-4x) > 0 \iff 1-4x > 0 \iff 1 > 4x$

$\iff \boxed{x < \frac{1}{4}}$. Since $e^{-4x} > 0$ for all x $f'(x) > 0$ only for $\boxed{x < \frac{1}{4}}$.

d. For what x values is $f(x)$ concave up?

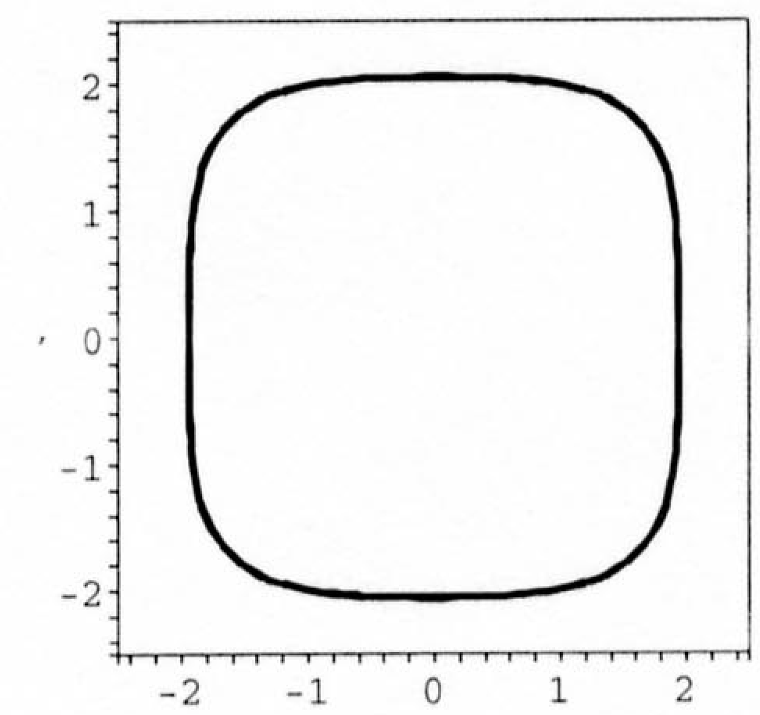
$f''(x) = 16xe^{4x} > 0$ for all ~~[illegible]~~ $x > 0$

hence f is concave up for ~~all~~ positive values of x.

5. *(16 points)* Let C be the curve which consists of the set of points for which

$$x^4 + x^2 + y^4 = 18$$

(see the graph at right).



Write the equation of the line tangent to C which passes through the point $(-1, -2)$.

Equation of a line passing through $(-1,-2)$ is

$$y - (-2) = m(x - (-1))$$

$$y + 2 = m(x+1).$$ So all we need is m.

Find it as follows: 1) differentiate both part of the equation $x^4 + x^2 + y^4 = 18$. and get

$$4x^3 + 2x + 4 \cdot y^3 \cdot y' = 0$$

$$\Rightarrow \quad y' = -(4x^3 + 2x) / 4y^3$$

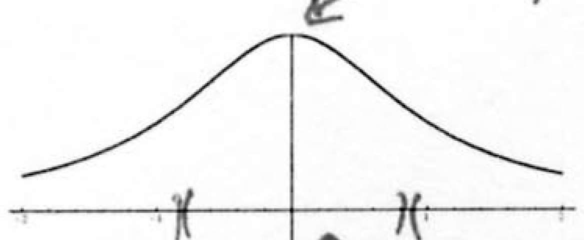
2) now plug $(-1,-2)$ in & get

$$y' = \frac{-(4\cdot(-1)^3 + 2(-1))}{4\cdot(-2)^3} = -\frac{-4-2}{4\cdot(-8)} = -\frac{6}{4\cdot 8} = \boxed{-\frac{3}{16}}$$

$$\Rightarrow \quad \boxed{y = -\frac{3}{16}(x+1) - 2}$$

6. *(16 points)* For each of the 4 functions graphed in the left column, find the corresponding derivative function among any of the 8 choices on the right (not just on the same row) and put its letter in the corresponding box.

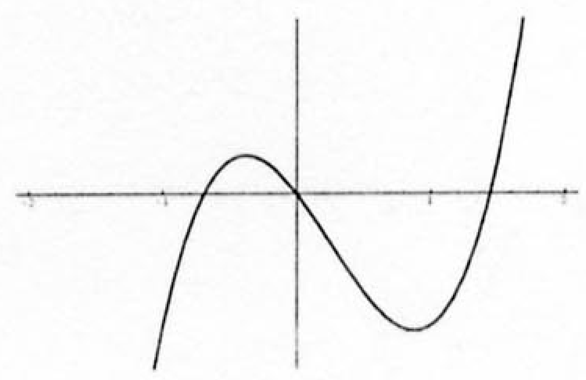
$f'(0)=0$

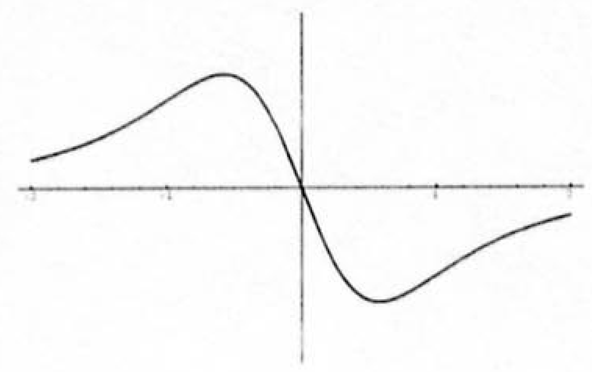
$\boxed{B}$ 

f' decreasing

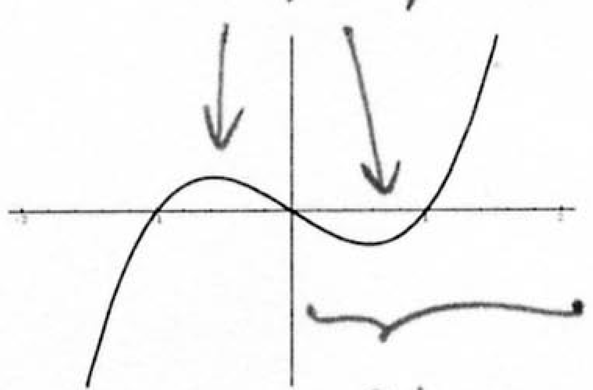
f' increasing

f is concave up.

A 

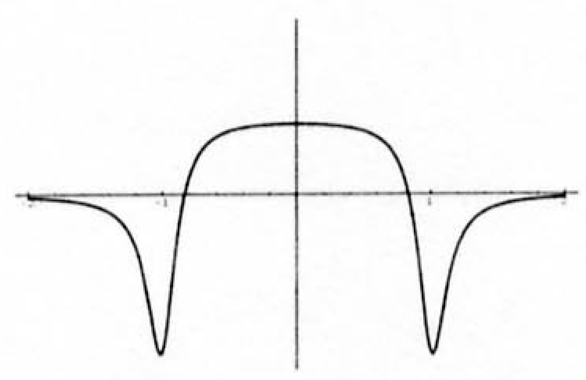
B 

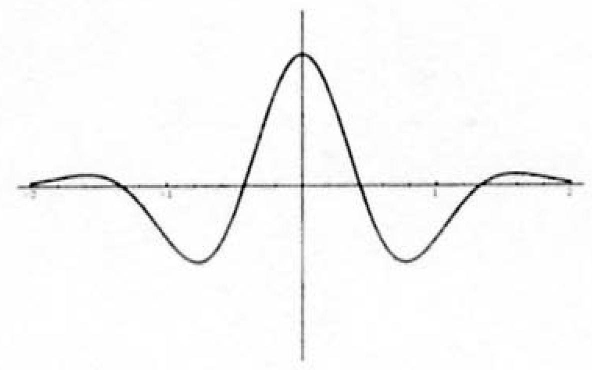
$f'(x)=0$

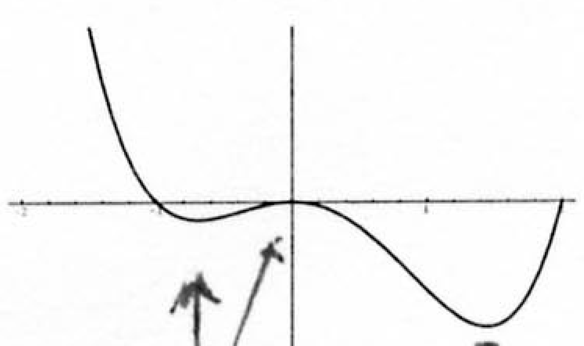
$\boxed{G}$ 

f' increasing

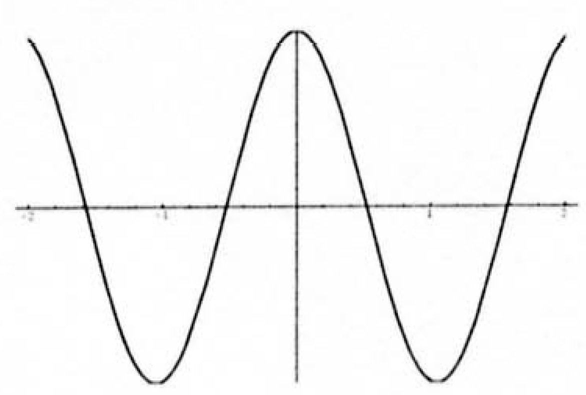
f' decreasing

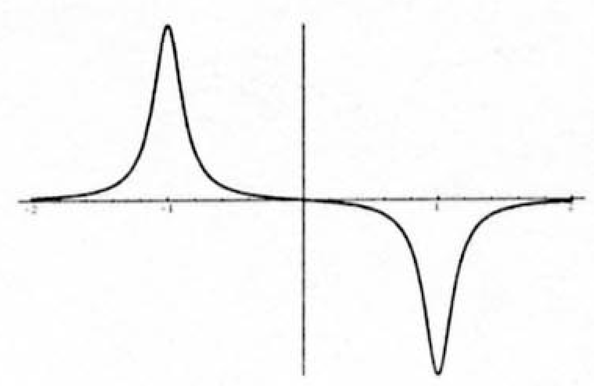
C 

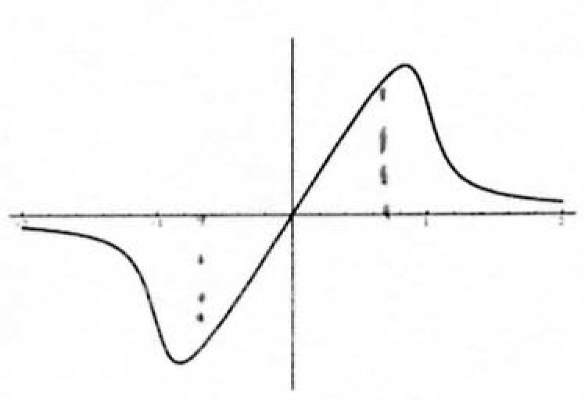
D 

$\boxed{A}$ 

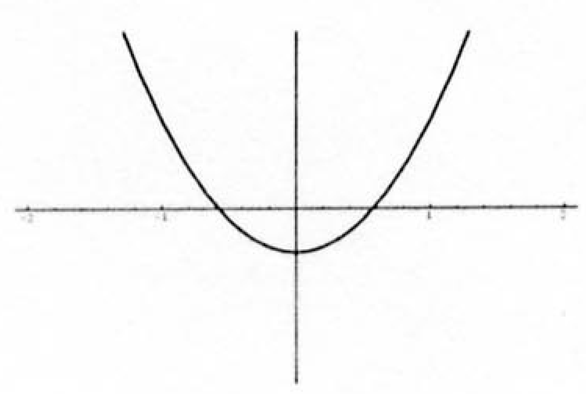
$f'(x)=0$.

E 

F 

$\boxed{C}$ 

$f' =$ const.

G 

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