Second Midterm Exam MAT 125, Spring 1999

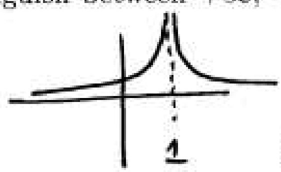
problem	1	2	3	4	5	6	7	Total
possible	16	12	16	20	10	6	20	100
score								

Name: **Section:**

Directions: There are 7 problems on five pages in this exam. Make sure that you have them all. Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate what is where if you expect someone to look at it. Books, extra papers, and discussions with friends are not permitted.

1. *(16 points)* Determine whether the following limits exist. If they do, find them. If the limit does not exist, distinguish between $+\infty$, $-\infty$, and no limiting behavior (DNE). Justify your answers.

a. $\lim_{x\to 1} \frac{2}{(x-1)^2}$

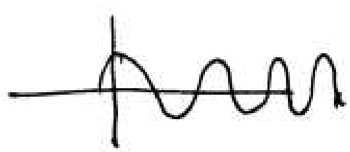


THE NUMERATOR REMAINS CONSTANT WHILE AS $X \to 1$, THE DENOMINATOR GETS ARBITRARILY SMALL, BUT ALWAY POSITIVE. THUS THE LIMIT IS $\boxed{+\infty}$

(SEE GRAPH)

b. $\lim_{x\to\infty} \frac{2x^3 - 7x + 1}{4x^3 + 3x^2 - 1} = \lim_{x\to\infty} \frac{2x^3}{4x^3} = \lim_{x\to\infty} \frac{2}{4} = \boxed{\frac{1}{2}}$

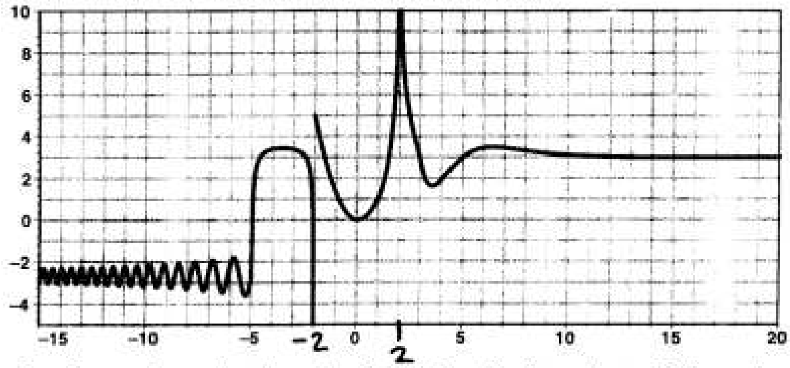
c. $\lim_{x\to\infty} \cos(x)$



COS(X) OSCILLATES AS $X \to \infty$, SO THERE CAN BE NO LIMIT $\boxed{\text{DNE}}$

d. $\lim_{x\to\infty} \sin(1/x) = \sin\left(\lim_{x\to\infty} \frac{1}{x}\right) = \sin(0) = \boxed{0}$.

2. *(12 points)* The following is the graph of a function $f(x)$.



Using the graph, determine whether the following limits exist. If they do, calculate them. If the limit does not exist, distinguish between $+\infty$, $-\infty$, and no limiting behavior (DNE).

a. $\lim_{x\to 2} f(x)$ $= +\infty$

b. $\lim_{x\to -2} f(x)$ DNE (SINCE $\lim_{x\to -2^-} f(x) = -\infty$ BUT $\lim_{x\to -2^+} = 5$)

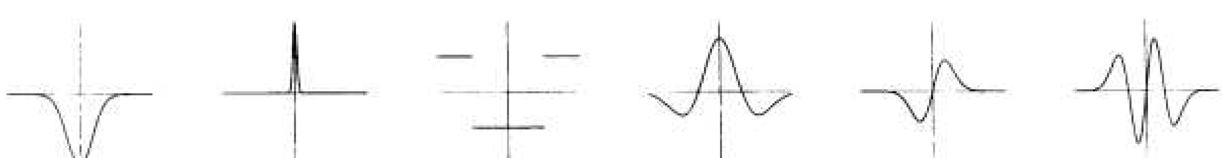
c. $\lim_{x\to -2^-} f(x)$ $= -\infty$

d. $\lim_{x\to\infty} \dfrac{f(x)}{x}$ (You may assume that for x large, $|f(x)| < 4$).

$0 = \lim_{x\to\infty} \frac{-4}{x} \le \lim_{x\to\infty} \frac{f(x)}{x} \le \lim_{x\to\infty} \frac{4}{x} = 0$, SO BY SQUEEZE THM, THE LIMIT IS $\boxed{0}$

3. *(16 points)* Below are the graphs of four functions, labeled (a) through (d). Underneath each, write the number of the graph (1) through (6) which corresponds to its derivative.

a 3 **b** 5 **c** 4 **d** 2



~~1~~ 2 3 4 5 ~~6~~

4. *(20 points)* For each function $f(x)$ below, find its derivative $f'(x)$.

a. $f(x) = x^2 e^x$ USE PRODUCT RULE !

$$\boxed{f'(x) = 2x e^x + x^2 e^x} = e^x(2x + x^2)$$

b. $f(x) = 5e^x - 2x^2$

$$\boxed{f'(x) = 5e^x - 4x}$$

c. $f(x) = \dfrac{x}{x^2+1}$ USE QUOTIENT RULE :

$$f'(x) = \frac{1 \cdot (x^2+1) - x(2x)}{(x^2+1)^2} = \frac{1 - x^2}{(x^2+1)^2}$$

d. $f(x) = e^{3\sqrt{x}}$ CHAIN RULE :

$$f'(x) = e^{3\sqrt{x}}\left(\frac{3}{2\sqrt{x}}\right)$$

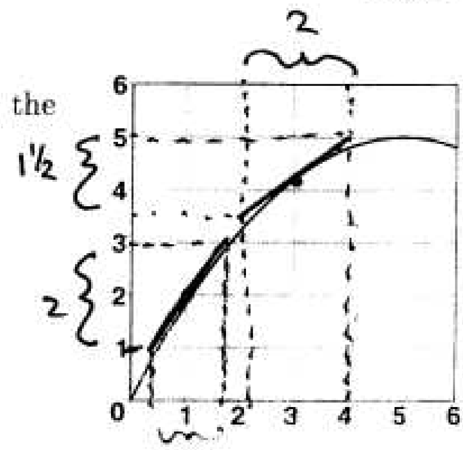
e. $f(x) = \left(\dfrac{x-3}{x+2}\right)^2$ CHAIN RULE & QUOTIENT RULE :

$$f'(x) = 2\left(\frac{x-3}{x+2}\right)\left(\frac{(x+2) - (x-3)}{(x+2)^2}\right) = \frac{10(x-3)}{(x+2)^3}$$

5. *(10 points)* At right is a graph of a function $f(x)$. Arrange the following quantities in increasing order:

$$1, f(0), f'(1), f'(3), f''(2).$$

Justify your answer.



$f(0) = 0$

$f'(1) \approx \mathbf{2}$ (TANGENT GOES UP 2 UNITS, AND OVER JUST ABOUT 1)

$f'(3) \approx \frac{1\frac{1}{2}}{2} = 3/4$ (GOES UP 1½ UNITS GOES OVER 2 UNITS)

$f''(2) < 0$ BECAUSE GRAPH IS CONCAVE DOWN.

THUS, THE ORDERING IS

$$f''(2) < f(0) < f'(3) < 1 < f'(1)$$

6. *(6 points)* Let $f(x) = x^x$. Write a limit which represents $f'(2)$. (Note that you do **not** need to compute or simplify the limit).

BY DEFINITION,

$f(2) = 2^2 = 4$

$$f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h \to 0} \frac{(2+h)^{2+h} - 4}{h}$$

7. *(20 points)* Let $f(x) = e^{3x^2+1}$.
Notice that $f(x) = g(h(x))$, where $g(x) = e^x$ and $h(x) = 3x^2 + 1$.

a. Calculate $f'(x)$. USE CHAIN RULE:

$$f'(x) = e^{3x^2+1} \cdot 6x = 6x\, e^{3x^2+1}$$

b. Calculate $f''(x)$. USE ANSWER TO (a) + PRODUCT + CHAIN RULE:

$$\begin{aligned} f''(x) &= 6e^{3x^2+1} + (6x)\cdot\left(\tfrac{d}{dx} e^{3x^2+1}\right) \\ &= 6e^{3x^2+1} + (6x)\cdot\left(6x e^{3x^2+1}\right) \\ &= 6e^{3x^2+1} + 36x^2 e^{3x^2+1} \end{aligned}$$

c. For which values of x is f increasing? **Hint:** Remember that for any t, $e^t > 0$.

f IS INCREASING WHEN $f'(x) > 0$.
FROM (a), WE NEED $6x\, e^{3x^2+1} > 0$
SINCE $e^{3x^2+1} > 0$ FOR ANY x (BY HINT),
f INCREASES WHEN $6x > 0$, THAT IS, FOR $\boxed{x > 0}$

d. Write the equation of the line tangent to $f(x)$ at the point $(1, e^4)$.

BY (a), $f'(1) = 6e^{3+1} = 6e^4$, THE SLOPE OF THE TANGENT AT $x = 1$.

LINE IS $$y - e^4 = 6e^4(x - 1)$$

e. Is f concave up at $x = 1$? Justify your answer.

YES, f IS CONCAVE UP AT $x = 1$, SINCE

$$f''(1) = 6e^4 + 36e^4 > 0.$$