First Midterm Exam MAT 125, Spring 1999

problem	1	2	3	4	5	6	7	Total
possible	10	10	20	18	12	15	15	100
score								

Name: **Section:**

Directions: There are 7 problems on five pages in this exam. Make sure that you have them all. Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate what is where if you expect someone to look at it. Books, calculators extra papers, and discussions with friends are not permitted. Leave all answers in exact form (that is, do *not* approximate π, square roots, and so on.)

1. *(10 points)* Let

$$f(x) = \begin{cases} x^3 & x < 0 \\ \tan x & 0 \leq x < \frac{\pi}{4} \\ 1 & x \geq \frac{\pi}{4} \end{cases}$$

For which values of x is $f(x)$ continuous? Justify your answer.

2. *(10 points)* Let Q be a square, and let d be the length of its diagonal. Write a function which expresses the area of Q as a function of d, and state the domain of this function.

3. *(20 points)* Compute each of the limits below. If the limit does not exist, say so. Justify your answer in all cases.

a. $\lim_{x\to 2} xe^{x-2}$

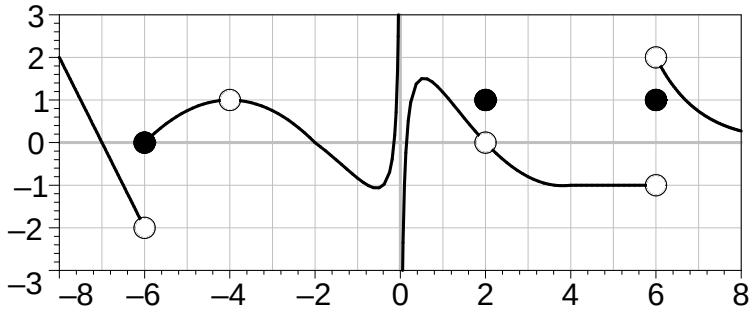
b. $\lim_{x\to 3} \dfrac{x^2-4x+12}{x-3}$

c. $\lim_{x\to 0+} x\cos(\ln x)$ Hint: recall that $-1 \le \cos x \le 1$ for any x.

d. $\lim_{x\to 0} \dfrac{(2-x)^2-4}{x}$

e. $\lim_{x\to \pi} \left(\sin x + e^{-\sqrt[7]{(x-\pi)^2}}\right)$

4.*(18 points)* Let $f(x)$ be the function whose graph is shown below.



a. Circle each value of x listed below where $f(x)$ is continuous from the right.

$-7 \quad -6 \quad -5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7$

b. List all points $-8 < x < 8$ where $f(x)$ is not continuous. If there are none, write "none".

c. What is $f(6)$? If it is not defined, write DNE.

d. What is $\lim_{x \to 2} f(x)$? If it is not defined, write DNE.

e. What is $\lim_{x \to 6^+} f(x)$? If it is not defined, write DNE.

f. What is $\lim_{x \to 4} \left(f(x/2) - f(-x)/2 \right)$? If it is not defined, write DNE.

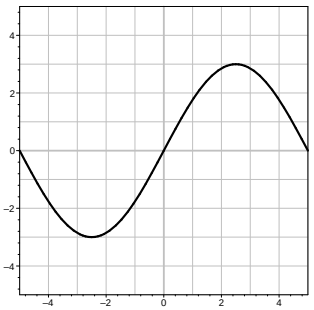
5. *(12 points)* Let $f(x) = \sqrt{\dfrac{x-1}{x}}$.

a. What is the domain of $f(x)$?

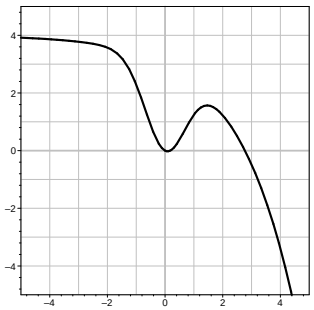
b. Find two functions g and h such that $f = g \circ h$.

c. Find a formula for $f^{-1}(x)$.

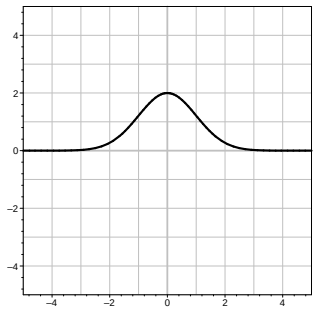
6. *(15 points)* The graphs of several functions $f(x)$ are shown below. On the same set of axes, sketch the function $g(x)$ as indicated.



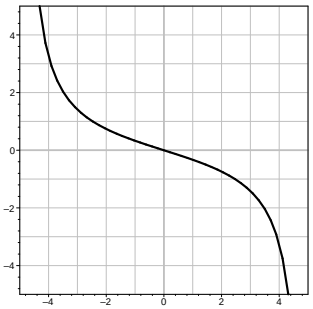
$g(x) = f(x/2)$



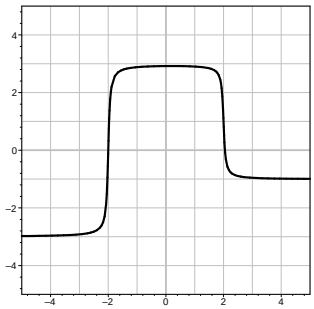
$g(x) = f(-x)$



$g(x) = 2 - 2f(x)$



$g(x) = f^{-1}(x)$



$g(x) = f(x+2)$

7.*(15 points)*

a. The values of the functions h and g are given by the table at right. What is the value of the function $h \circ g$ at 1?

x	g(x)	h(x)
0	1	2
1	0	1
2	2	0

b. If $5e^{2x} = 10$, what is x?

c. What is the domain of the function $\ln|x|$?

d. What is the inverse of $f(x) = x^2$, with $x < 0$? If the function is not invertible, write "no inverse".

e. Write the equation of the line passing through the point $(-2, 4)$ that has slope 3.