Key to Problem Set 8

1. Use Chain Rule, we have

$$\begin{aligned} f'(x) &= \frac{1}{x^6+5}(x^6+5)' \\ &= \frac{6x^5}{x^6+5} \end{aligned}$$

2. Still Chain Rule,

$$\begin{aligned} f'(\theta) &= \frac{1}{\cos 5\theta} \cdot (-\sin 5\theta) \cdot 5 \\ &= -5\tan 5\theta \end{aligned}$$

3. Chain Rule:

$$\begin{aligned} f'(x) &= \cos(\ln 11x) \cdot (\ln 11x)' \\ &= \cos(\ln 11x) \cdot \frac{11}{11x} \\ &= \frac{\cos(\ln 11x)}{x} \end{aligned}$$

4. Note that $f(x) = (\ln x)^{1/3}$, we have

$$\begin{aligned} f'(x) &= \frac{1}{3}(\ln x)^{1/3-1} \cdot (\ln x)' \\ &= \frac{1}{3} \cdot (\ln x)^{-2/3} \cdot \frac{1}{x} \\ &= \frac{1}{3x\sqrt[3]{(\ln x)^2}} \end{aligned}$$

So the second choice.

5. Use Quotient Rule,

$$\begin{aligned} y' &= \frac{\frac{1}{x}(3+x) - \ln x \cdot 1}{(3+x)^2} \\ &= \frac{3+x-x\ln x}{x(3+x)^2} \end{aligned}$$

So the first choice.

6. First we have

$$G(u) = \frac{1}{2}\ln(4u+4) - \frac{1}{2}\ln(4u+4),$$

thus

$$\begin{aligned} G'(u) &= \frac{1}{2}\frac{4}{4u+4} - \frac{1}{2}\frac{4}{4u-4} \\ &= \frac{1}{2}4(4u-4) - 4(4u+4)(4u+4)(4u-4) \quad \text{(common denominator)} \\ &= -\frac{16}{16u^2-16} \end{aligned}$$

The third choice.

7. $f'(x) = \cos(3x)\cdot 3 + \frac{1}{2x}\cdot 2 = 3\cos 3x + \frac{1}{x}$.

8. Take ln on both sides we have

$$\ln y = \ln x^{5x} = 5x\ln x,$$

by implicit differentiation, we get

$$\begin{aligned} \frac{1}{y}\cdot y' &= 5\ln x + 5x\frac{1}{x} \\ &= 5\ln x + 5 \end{aligned}$$

So

$$\begin{aligned} y' &= y(5\ln x + 5) \\ &= x^{5x}(5\ln x + 5) \end{aligned}$$

9. From the graph we see that $P(2030) \approx 21\%$. Draw the tangent line at $t = 2030$ and use this as linear approximation we get $P(2040) \approx 23\%$ and $P(2050) \approx 25\%$.

10. For (a)-(c), we have the following formula:

$$V = a^3 \quad \text{(a denotes the lengthe of the edge)}$$

differentiate the above function we get

$$\frac{dV}{da} = 3a^2$$

which means

$$dV = 3a^2 da$$

or we may write

$$\Delta V \approx 3a^2 \Delta a$$

So for (a), we have

$$\begin{aligned} \text{maximun possible error} &= \Delta V \\ &\approx 3a^2\Delta a \\ &= 3 \times 3 \times 40^2 \times 0.3 \\ &= 1440\ cm^3 \end{aligned}$$

(b). The volume of the cube is $V = a^3 = 64000\ cm^3$,

$$\begin{aligned} \text{relative error} &= \frac{\Delta V}{V} \\ &\approx \frac{1440}{64000} \\ &= 0.0225 \end{aligned}$$

(c). From (b), percentage error $\approx 2.25\%$.

Similarly for (d)-(e), we have

$$S = 6a^2$$

Thus

$$\Delta S = 12a\Delta a$$

So

$$\begin{aligned} \text{maximun possible error} &= \Delta S \\ &\approx 12a\Delta a \\ &= 12 \times 40 \times 0.3 \\ &= 144\ cm^2 \end{aligned}$$

(e). The surface area of the cube is $S = 6a^2 = 9600\ cm^2$. so

$$\begin{aligned} \text{relative error} &\approx \frac{\Delta S}{S} \\ &\approx \frac{144}{9600} \\ &= 0.015 \end{aligned}$$

(f).From (e), percentage error $\approx 1.5\%$.

11. The volume of a sphere is given by

$$V = \frac{4}{3}\pi r^3$$

So the volume of the hemisphere is

$$V_{hemi} = \frac{2}{3}\pi r^3$$

Differentiate the above we get

$$dV_{hemi} = 2\pi r^2 dr$$

We can use this to estimate the amount of paint needed:

$$\begin{aligned}
\Delta V_{hemi} &\approx 2\pi r^2 \Delta r \\
&= 2\pi(50)^2 \times 0.04 \times 10^{-2} \\
&= 2\pi \\
&\approx 6.283\ m^3
\end{aligned}$$