Solutions to Web-Based Homework 3

This is the 'semi-detailed' solution set for the third web-based homework. Hopefully you will find it useful and instructive. For now and in the future, these solutions will be available only after the web-based homework is due. Hopefully, I'll be sending you the solutions the day after each set is due.

Please take advantage of these solutions, the recommended homework problems in your text, my office hours, and our recitation. Based on the averages of the first midterm, we as a class need to strengthen the basics while moving on to other topics in calculus.

Problem 1:

The answer to this question is simple. All that you need to do, is find the slope (rate of change) of the line, determined by the two points specified. However they only give you the$x - values$ of the two points, you must use the table to determine the corresponding $y - values$ of the two points. Hence you will be able to calculate the slope of the line, and (if you wanted to) then the $y - intercept$, and write an equation of the form, $y = mx + b$.

So, at $t = 38$, $H(t) = 2700$, and at $t = 42$, $H(t) = 2960$. So the slope will be:

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

$$m = \frac{2960 - 2700}{42 - 38} = \frac{260}{4} = \frac{130}{2} = 65$$

Thus the answer is 65 hearbeats per minute.

Problem 2:

This problem asks you to find the equation of a line, which is tangent to a curve, P, at some point on P. The information given tells you that the line is tangent at the point $(2, 22)$, which is on the graph of P. They also tell you that the slope of this tangent line is 10. So this makes life easy, all we need to do is find the $y - intercept$ of the line, and write the equation of the tangent line:

$$y = mx + b.$$

In fact we know that the equation of the line is: $y = 10x + b$. Now all we need to do is 'plug in' the point, $(2, 22)$ and we will have found b. So,

$$22 = 10 * 2 + b$$

$$22 - 20 = b$$

$$b = 2$$

So

$$y = 10x + 2.$$

Problem 3:

To find the average velocity, one must use a formula like the one below:
If $v(t)$ is some velocity function, then the average velocity between some time $t = \mu$ and a later time $t = \nu$ is:

$$\frac{v(\mu) - v(\nu)}{\mu - \nu}$$

So using the equation, one can calculuate what the average velocity is of the given function, between $t = 3$ and $t = 3.2$ and $t = 3$ and $t = 3.02$.
The given velocity function is:

$$y = 35t - 13t^2$$

Now,

$$y(3) = 35 * 3 - 13 * (3)^2 = -12ft/s$$

and

$$y(3.2) = 35 * (3.2) - 13 * (3.2)^2 = -21.12ft/s$$

So,

$$\frac{y(3.2) - y(3)}{3.2 - 3} = \frac{-21.12 - (-12)}{3.2 - 3} = \frac{-9.12}{.2} = -45.6$$

And, if we replace μ with 3.02 while keeping $\nu = 3$, then the average velocity will be:

$$\frac{y(3.02) - y(3)}{3.02 - 3} = \frac{-12.8652 - (-12)}{3.02 - 3} = \frac{-.8652}{.02} = -43.26.$$

These are the respective answers for the average velocities, for $3 \leq t \leq 3.2$, and $3 \leq t \leq 3.02$.

Problem 4:

The answer to this problem is to find the average velocity between $t = 1$ and $t = 2$ and then the average velocity between $t = 2$ and $t = 3$. These two numbers will be used to determine an approximation to the instantaneous velocity between $t = 1$ and$t = 3$. So, the average velocity between 1 and 2 is:

$$\frac{32 - 12}{2 - 1} = 20$$

and the average velocity between 2 and 3 is:

$$\frac{78 - 32}{3 - 2} = 46.$$

Now, the approximate instantaneous velocity between 1 and 3 is:

$$\frac{78 - 32 - (32 - 12)}{3 - 2 - (2 - 1)} = \frac{66}{2} = 33$$

And this is the answer.

Problem 5:

If

$$\lim_{x \to 5^-} f(x) = 4$$

and if

$$\lim_{x \to 5} f(x) = L$$

i.e. if the limit exists as x approaces 5, this tells us that the limit of $f(x)$ as x approaches 5 from the right equals the limit of $f(x)$ as x approaches 5 from the left. So

$$\lim_{x \to 5} f(x) = 4.$$

As x approaches 5 the limit must be 4. This is all based on what the definition of the "total" or "two-sided" limit is. A function has a "two-sided limit" at some point x_0 if the limit from the right and left, while approaching x_0

equal each other. Hence if the "two-sided" limit exists, then the right and left sided limits equal each other.

Problem 6:

The limit of $f(x)$, as x approaches 1 from the right is 1.5. This is found by following the graph from the right towards $x = 1$, and looking at the $y - value$ of the function at $x = 1$. By inspection, one can see that the $y - value$ from the right will be 1.5.

Problem 7:

This problem is very similar to the previous one, problem 6. The idea is exactly the same, except for the fact that now, one must follow the graph from the left and move towards $x = 1$, and then look at the corresponding $y - value$ of the function. If one does this, it is quite clear that the limit of $f(x)$ as x approaches 1 from the left $(x \to 1^-)$ is 2. By inspection, one can see that this is the $y - value$ that the function approaches.

Problem 8:

The correct graph for the is piece-wise function is graph 3 (the last one). This is somewhat clear by inspection as well. The graph $f(x)$ is the line $y = x$ for $-1 \leq x < 2$. However for $x \geq 2$ the function looks like the parabola $y = x^2$, but the parabola is shifted 2 units to the right, since there is a -2 inside the parentheses. Finally, for $x < -1$ the function looks like the line $y = 2 - x$. Clearly, the only graph that satisfies even the first part (that between -1 and 2 the function looks like the line $y = x$) is the last one.

Problem 9:

The answer to this problem can be found in two ways. The first way is simpler, but only works since we saw the graph of this function in the previous problem (the second way, involves one graphing the function himself, or analytically looking at the endpoints of the intervals of the "partial domains" of the functions which comprise the piece-wise function and seeing if the limits from the left and right mathc up: e.g. looking if $f(x) = x$ matches $f(x) = (x - 2)^2$ at $x = 2$). The first method only requires one to inspect the graph of the function and then determine the $x - values$ of the function for which left and right limits do not match. Those $x - values$ are the values for which the limit of $f(x)$ does not exist for arbitrary a. The subsequent $x - values$ for which limits exist for the function $f(x)$ are:

$$(-\infty, -1) \cup (-1, -2) \cup (2, \infty).$$

The reason that these are the $x - values$ at which limits exists for $f(x)$ is simply because these are $x - values$ for which the left and right sided limits match (i.e. they equal each other). If one is clever, he would notice that the end points of each of these intervals is essentially the endpoints of the intervals which specify the different types of functions which comprise the piece-wise function $f(x)$. This is not the case always. It is the case here, because the piece-wise functions do not "connect" to one another at the end-points of the intervals for which they are defined. For example, for $x \geq 2$, $f(x) = (x-2)^2$,and for $-1 \leq x < 2$ $f(x) = x$. Hence, the limit of $f(x)$ as x approaches 2 from the left and right do not equal each other. From the left the limit of $f(x)$ equals 2, and from the right the limit of $f(x)$ equals 0.

Problem 10:

When I went online, my computer wouldn't show me the choices for the graphs. So I can't give you the answer to this one. Hopefully, I will have covered the answer in class, if someone asks for it.

Problem 11:

The

$$\lim_{x \to 1} f(x) = 3 * \frac{x-1}{x^3-1} = 1$$

as $x \to 1$, $f(x)$ is 1. The answer is found by looking at the table, which indicates the behavior of the function $f(x)$, as x gets closer and closer to 1.

Problem 12:

The method of finding the answer to this problem is exactly the same as that of the previous problem. All one needs to do is look at the behavior of the function as $x \to 3$. This behavior is given by the table. As x approaches

3, $f(x)$ approaches .333333. That is,

$$\lim_{x \to 3} \frac{x + e^3 - e^x}{x^2} = .333333$$

Problem 13:

There is one sure-fire method by which to calculate limits (without the more advanced techniques which you are going to learn in the next month or so, in this course). An easy way to check if a limit exists, and if it does, what it's value is, is to appoximate the function at the limit value desired. For example in this case, the function is:

$$f(x) = \frac{tan(2x)}{x}.$$

And we want to calclulate

$$\lim_{x \to 0} \frac{tan(2x)}{x}.$$

As one can see, we cannot directly calculate this limit, by "plugging in" 0 for x, since that would yield a 0 in the denominator. However, if we look at $f(0.1)$ and $f(-0.1)$, and then $f(0.01)$, and $f(-0.01)$, etc. we can see if the limit exists, and if it does, what its value is.

Finding $f(0.1)$, $f(0.01)$, and $f(0.001)$, etc, is essentially finding:

$$\lim_{x \to 0^+} f(x).$$

And finding $f(-0.1)$, $f(-0.01)$, and $f(-0.001)$, etc. is essentially finding:

$$\lim_{x \to 0^-} f(x)$$

Obviously if the numnbers equal each other (roughly) then we know the limit exists and equals a number close to the numbers found.

So, $f(0.1) = 2.0271$, and $f(-0.01) = 2.0271$. Also, $f(0.01) = 2.0002$, and $f(-0.01) = 2.0002$. Clearly, the limit is 2. So,

$$\lim_{x \to 0} \frac{tan(2x)}{x} = 2.$$

Problem 14:

Finding the slope of the tangent line of $f(x) = 3^x$ at $(0, 1)$ is the same as finding the following limit (if you don't understand why the following limit is the slope, that's OK. We'll talk about that this week):

$$\lim_{x \to 0} \frac{3^x - 1}{x}.$$

Once again, we cannot directly "plug in" 0 for x, since it would yield a 0 in the denominator. So we must use the same technique that was used above. So, we should find $f(0.1)$, $f(0.01)$, $f(-0.1)$, and $f(-0.01)$. So $f(0.1) = 1.1612$, $f(0.01) = 1.1046$, $f(-0.1) = 1.0404$, and $f(-0.01) = 1.0925$. This clearly indicates that the limit is approching somthing like 1.09. So the estimate for the limit is 1.09.

Problem 15:

I will only show the numerical aspect of the estimation of this limit, since it is difficult for me to put graphs into this type of document. The method of numerical estimation is exactly the same as that of problem 13 and problem 14. So let's proceed.

If

$$f(x) = \frac{x^3 - 64}{4(\sqrt{x} - 2)},$$

then we want to compute the following limit:

$$\lim_{x \to 4} \frac{x^3 - 64}{4(\sqrt{x} - 2)}.$$

Let's approximate this limit from both sides by finding $f(3.9)$, $f(3.99)$, $f(4.1)$, and $f(4.01)$. This should be enough to characterize the behavior of this function at $x = 4$. So, $f(3.9) = 46.5155$, $f(3.99) = 47.8501$, $f(4.1) = 49.5156$, and $f(4.01) = 48.1501$. So the limit is 48.