

Homework II.

① Find $f \circ g \circ h$.

$$f(x) = \sqrt{x-4}$$

$$g(x) = x^2 + 5$$

$$h(x) = x + 1$$

$$\begin{aligned} \Rightarrow (f \circ g \circ h)(x) &= \sqrt{(x+1)^2 + 5 - 4} \\ &= \sqrt{x^2 + 2x + 2} \end{aligned}$$

② Find g if $F = f \circ g$.

$$F(x) = (x^2 + 4)^{12}$$

$$\Rightarrow g(x) = x^2 + 4$$

③ Find h , if $H = f \circ g \circ h$

$$H(x) = 4 \cdot 3^{x^2}$$

$$\Rightarrow h(x) = x^2$$

$$\textcircled{4} \quad \left. \begin{array}{l} g(3) = 3 \\ f(3) = 1 \end{array} \right\} \Rightarrow (f \circ g)(3) = 1.$$

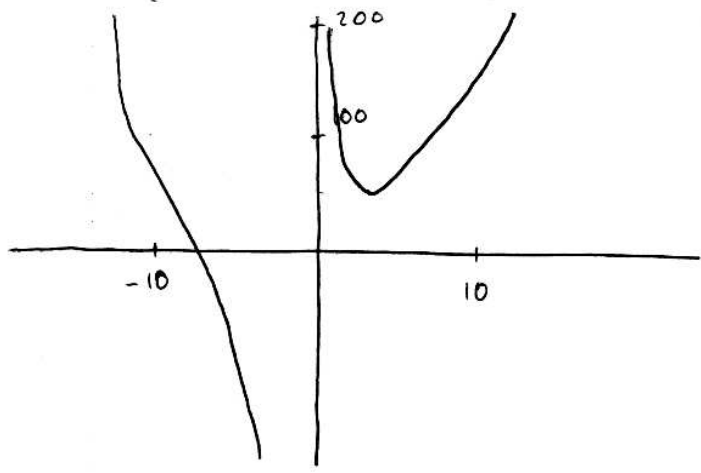
$$\textcircled{5} \quad f(x) = x + 5$$

$$h(x) = 4x - 5$$

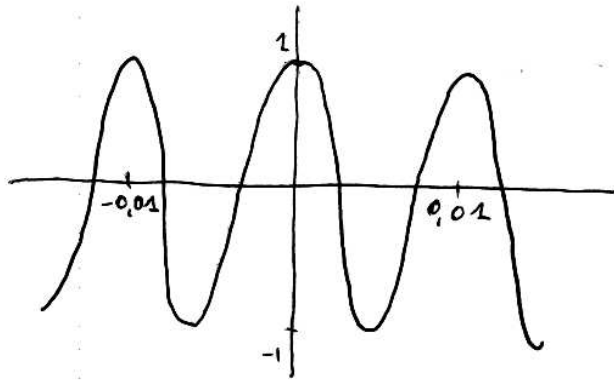
Find g such that $g \circ f = h$.

$$g(f(x)) = h(x) \Rightarrow g(x+5) = 4x - 5 \Rightarrow g(x) = 4x - 25$$

⑥ Graph of $f(x) = x^2 + \frac{180}{x}$



⑦ Graph $f(x) = \cos 600x$



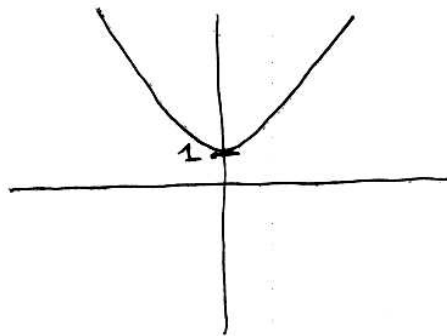
⑧ $f(x) = 7x^2$

$$g(x) = \frac{x^3}{7}$$

When x is very large ($x > 49$) $g(x) > f(x)$.

⑨ $f(x) = \sqrt{1+cx^2}$

$$\Rightarrow c > 0$$



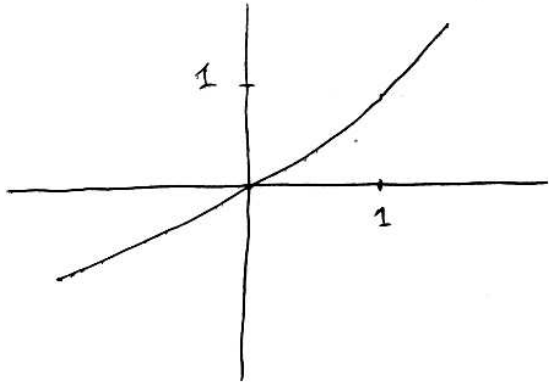
$$\textcircled{10} f(x) = 2^x$$

$$\Rightarrow f(-1) = \frac{1}{2}, f(0) = 1, f(1) = 2, f(2) = 4.$$

$$\textcircled{11} f(x) = 3^{-x}$$

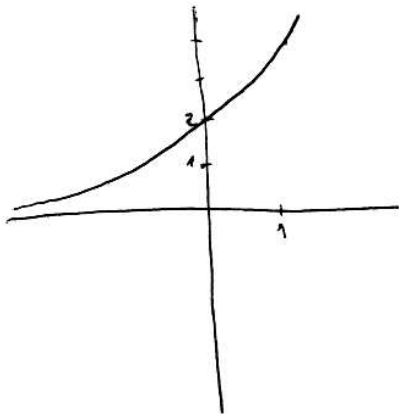
$$\Rightarrow f(-1) = 3, f(0) = 1, f(1) = \frac{1}{3}, f(2) = \frac{1}{9}.$$

$$\textcircled{12} f(x) = 2^x - b.$$



$$\Rightarrow b = 1$$

$$\textcircled{13} f(x) = ba^x$$



$$\Rightarrow f(x) = 2 \cdot 2^x$$

⑭ 2 is better than 1.

$$\textcircled{15} \quad 2^3 \cdot 60 = 480.$$

⑮ The given function is one-to-one.

⑯ f is one-to-one, $f(8) = 1 \Rightarrow f^{-1}(1) = 8.$

$$\textcircled{18} \quad f(x) = 7 + x^2 + \tan \pi x \quad \text{where } -1 < x < 1.$$
$$f^{-1}(7) = 0$$

$$\textcircled{19} \quad y = \ln(5x+6)$$

$$\Rightarrow e^y = 5x+6$$

$$\Rightarrow x = \frac{e^y - 6}{5}$$

$$\Rightarrow f^{-1}(x) = \frac{e^x - 6}{5}$$

$$\textcircled{20} \quad y = \frac{10 + e^x}{10 - e^x}$$

$$\Rightarrow y(10 - e^x) = 10 + e^x$$

$$\Rightarrow -e^x - ye^x = 10 - 10y$$

$$\Rightarrow e^x(-1-y) = 10(1-y)$$

$$\Rightarrow e^x = \frac{10(1-y)}{-1-y}$$

$$\Rightarrow x = \ln\left(\frac{10(1-y)}{-1-y}\right)$$

$$\Rightarrow f^{-1}(x) = \ln\left(\frac{10(1-x)}{-1-x}\right)$$

$$\textcircled{21} \text{ solve } 8 \ln x = 7$$
$$\Rightarrow \ln x = \frac{7}{8}$$
$$\Rightarrow x = e^{7/8}$$

$$\textcircled{22} \text{ solve } \ln \ln x = 6$$
$$\Rightarrow \ln x = e^6$$
$$\Rightarrow x = e^{e^6}$$

$$\textcircled{23} \text{ solve } 50 \cdot 2^{t/6} = 90000$$
$$\Rightarrow 2^{t/6} = 1800$$
$$\Rightarrow e^{t/6 \ln 2} = 1800$$
$$\Rightarrow \frac{t}{6} \ln 2 = \ln 1800$$
$$\Rightarrow t = \frac{6 \ln 1800}{\ln 2}$$
$$\Rightarrow t = 64,88.$$