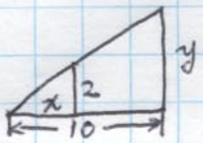
Homework 10 Dezhen Xu 12/5/2004

1. $\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt} = (3x^2+5)\frac{dx}{dt} = 320$

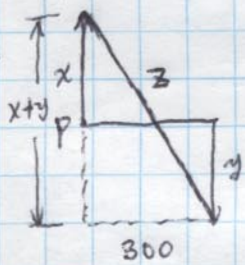
2. 

$\frac{dx}{dt} = 1.4\ m/s$

By similar triangles, $\frac{y}{10} = \frac{2}{x} \Rightarrow y = \frac{20}{x}$

$\Rightarrow \frac{dy}{dt} = -\frac{20}{x^2}\frac{dx}{dt} = -\frac{20}{x^2}(1.4)$. When $x = 5m$,

$\frac{dy}{dt} = -1.12\ m/s$. so the shadow is decreasing at a rate of $1.12\ m/s$.

3. 

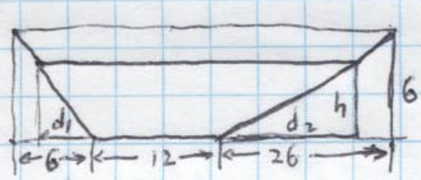
We have $\frac{dx}{dt} = 6\ ft/s$, $\frac{dy}{dt} = 3\ ft/s$.

$z^2 = (x+y)^2 + 300^2 \Rightarrow 2z\frac{dz}{dt} = 2(x+y)\left(\frac{dx}{dt}+\frac{dy}{dt}\right)$

20 min after the woman starts, we have

$$\left.\begin{array}{l} x = (6ft/s)(25\ min)(60s/min) = 9000\ ft \\ y = (3ft/s)(20\ min)(60s/min) = 3600\ ft \end{array}\right\} \Rightarrow z = 100\sqrt{15885}\ ft$$

So $\frac{dz}{dt} = \frac{x+y}{z}\left(\frac{dx}{dt}+\frac{dy}{dt}\right) = \frac{9000+3600}{100\sqrt{15885}}(6+3) \approx 9.00\ ft/s$

4. 

$\frac{h}{6} = \frac{d_1}{6} = \frac{d_2}{26} \Rightarrow$

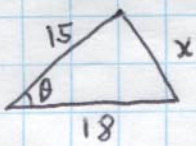
$V = \frac{1}{2}(12+12+d_1+d_2)h w$

$= \frac{1}{2}(24 + h + \frac{26}{6}h)h \cdot 14$

$= (12 + \frac{13}{6} + h)h \cdot 14$

$\Rightarrow \frac{dV}{dt} = \frac{dh}{dt}(14h) + (12+\frac{13}{6}+h)14 \cdot \frac{dh}{dt} \xlongequal{h=5} \frac{145*7}{3}\frac{dh}{dt}$

$$\Rightarrow \frac{dh}{dt} = \frac{dV}{dt} * \frac{3}{145*7} = 0.8 * \frac{3}{145*7} \approx 0.00148 \text{ ft/min}$$

5. 

What we know: $\frac{d\theta}{dt} = 4°/\text{min} = \frac{\pi}{45}$ rad/min. What we want: $\left.\frac{dx}{dt}\right|_{\theta=60°} = ?$

By the Law of Cosines,

$$x^2 = 15^2 + 18^2 - 2(15)(18)\cos\theta = 549 - 540\cos\theta$$

$$\Rightarrow 2x\frac{dx}{dt} = 540\sin\theta\frac{d\theta}{dt}$$

$x = \sqrt{549 - 540\cos 60°}$ when $\theta = 60°$

$= 3\sqrt{31}$

$$\Rightarrow \left.\frac{dx}{dt}\right|_{\theta=60°} = \frac{(540)(\frac{\sqrt{3}}{2})\frac{\pi}{45}}{2(3\sqrt{31})} \simeq 0.977 \text{ m/min}$$

6. With $R_1 = 85$ and $R_2 = 135$, $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{85} + \frac{1}{135} = \frac{44}{135*17}$.

Differentiating $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ w.r.t. t, we have

$$-\frac{1}{R^2}\frac{dR}{dt} = -\frac{1}{R_1^2}\frac{dR_1}{dt} - \frac{1}{R_2^2}\frac{dR_2}{dt}$$

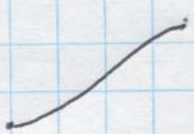
$$\Rightarrow \left.\frac{dR}{dt}\right|_{\substack{R_1=85\\R_2=135}} = \frac{27^2}{44^2}(0.3) + \frac{17^2}{44^2}(0.7) \approx 0.217 \ \Omega/s$$

7. By definition, abs. max: $\underline{f(4)=5}$, local max: $\underline{f(1)=4,\ f(4)=5}$

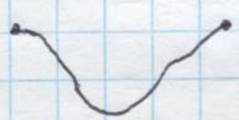
abs. min: $\underline{f(2)=1,\ f(8)=1}$, local min: $\underline{f(2)=1}$.

8.

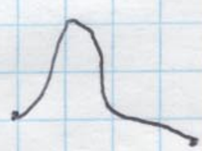
abs. max at 5.
abs. min at 1



abs. max at 1 and 5.
abs. min at 3. loc. min at 3



abs. max at 2.
abs. min at 5.
local max at 2.



9. $0 = f'(x) = 16x + 4 \Rightarrow x = \frac{-1}{4}$

10. $0 = f'(x) = 3x^2 + 10x - 48 \Rightarrow x = -6, \; x = \frac{8}{3}$

11. $G'(x) = \frac{1}{3}(x^2 - 2x)^{-\frac{2}{3}}(2x-2) \Rightarrow$ critical numbers : 2, 1, 0.

12. $f'(x) = 2\sin(5x) \cdot \cos(5x) \cdot 5 \Rightarrow$ critical numbers : $x = \frac{\pi n}{10}$, n integers.

13. $F'(x) = \frac{2}{5} x^{-\frac{3}{5}} (x-7)^2 + x^{\frac{2}{5}} \cdot 2(x-7) = 0$

$\Rightarrow$ critical numbers : $x = 0, \; x = 7$, or $x = \frac{7}{6}$

14. $f(x) = 5xe^{-x}, \Rightarrow f'(x) = 5e^{-x} - 5xe^{-x} = 5e^{-x}(1-x)$

so, $f'(x)$ is increasing on $[0,1]$, decreasing on $[1,2]$,

$\Rightarrow$ absolute max. value $f(1) = \frac{5}{e}$.

15. $f'(x) = 5 \cdot \frac{\frac{1}{x} \cdot x - \ln x}{x^2} = 5 \frac{1 - \ln x}{x^2} = 0 \Rightarrow x = e$

$f(1) = 0, \; f(3) = 5\frac{\ln 3}{3}, \; f(e) = \frac{5}{e} \Rightarrow$ absolute max value : $\frac{5}{e}$

16. $f'(x) = 2x - 6 \Rightarrow f(x) \nearrow$ on $[3, 5]$, $f(x) \searrow$ on $[2,3]$.

$f(2) = 1, \; f(3) = 0, \; f(5) = 4 \Rightarrow \begin{cases} \text{abs. max : } f(5) = 4 \\ \text{abs. min : } f(3) = 0 \end{cases}$

17. Similarly as above $\Rightarrow$ abs. max : $f(4\sqrt{2}) = 64$

abs. min : $f(-5) = -10\sqrt{39}$.

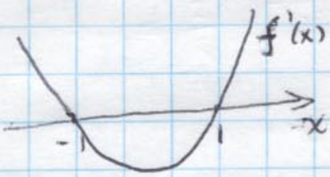
18. Similarly $\Rightarrow$ absolute max : $f(0) = 5$.

19. Similarly $\Rightarrow f(\pi) = -1, \; f(\frac{5\pi}{4}) = -\sqrt{2}$.

20. By definition, inflection points are where $f''(x)$ changes signs.

$\Rightarrow x = 2, 10$.

21. $f'(x) = 3x^2 - 3$ $\qquad$ $f'(x) > 0 \Leftrightarrow f \nearrow$, $f'(x) < 0 \Leftrightarrow f \searrow$



$\Rightarrow$

(a) $(-\infty, -1), (1, \infty)$.

(b) $(-1, 1)$

22.

$y' = 3ax^2 + 2nx + p$, $a \neq 0$

has at most 2 roots $\Rightarrow$ the maximum quantity of local extreme values of cubic function is 2.