MAT125 HW1

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1. complete the table if
$$f(x) = 3x^2 - 5x + 2$$
Give your answer as a number, not an expression.

x	$f(x)$
3	
-3	
4	
-4	

Solution Reminder: when plugging in negative numbers or expressions into x, always use paranthesis!

$$f(3) = 3 \cdot 3^2 - 5 \cdot 3 + 2 = 27 - 15 + 2 = 14$$

$$f(-3) = 3 \cdot (-3)^2 - 5 \cdot (-3) + 2 = 27 + 15 + 2 = 44$$

$$f(4) = 3 \cdot 4^2 - 5 \cdot 4 + 2 = 48 - 20 + 2 = 30$$

$$f(-3) = 3 \cdot (-4)^2 - 5 \cdot (-4) + 2 = 48 + 20 + 2 = 70$$

So here we complete the table:

x	$f(x)$
3	14
-3	44
4	30
-4	70

2. A spherical balloon with radius r inches has volume $\frac{4}{3}\pi r^3$.
Find a function that represents the amount of air required to inflate the balloon from a radius of r inches to a radius of $r + 5$ inches.
Solution Replacing r in the formula $\frac{4}{3}\pi r^3$ by $r + 5$, we get the volume of the sphere with radius $r + 5$. So here is the situation:
The volume of the sphere of radius $r : V_1 = \frac{4}{3}\pi r^3$
The volume of the sphere of radius $r + 5 : V_2 = \frac{4}{3}\pi(r + 5)^3$
The amount of air required to inflate the balloon from volume V_1 to V_2 is the difference of these two volumes, i.e.

$$V_2 - V_1 = \frac{4}{3}\pi(r + 5)^3 - \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left((r + 5)^3 - r^3\right)$$

(Here I am factoring out $\frac{4}{3}\pi$ by the distribution law.)
Let's simplify $(r+5)^3$ first :

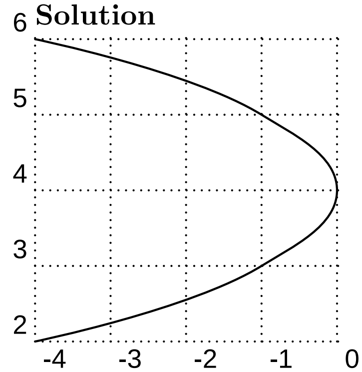
$$(r+5)^3 = (r+5)\cdot(r+5)\cdot(r+5) = (r\cdot r + r\cdot 5 + 5\cdot r + 5\cdot 5)\,(r+5) = (r^2+10r+25)(r+5) = (r^2\cdot r+r^2\cdot 5+$$

So the answer is:

$$V_2 - V_1 = \frac{4}{3}\pi\left((r+5)^3 - r^3\right) = \frac{4}{3}\pi(r^3+15r^2+75r+125-r^3) = \frac{4}{3}\pi(15r^2+75r+125)$$

3. Find an expression for the function $y = f(x)$ whose graph is the bottom half of the parabola $x + (4-y)^2 = 0$.

Solution



Looking at the graph, we see that the 'bottom half of the graph' means the part of graph where $y < 4$. Keeping this in mind, let us solve $x + (4-y)^2 = 0$ by y

$$(4-y)^2 = -x$$
$$(4-y) = \pm\sqrt{-x}$$
$$y = 4 \pm \sqrt{-x}$$

So we have two expressions for y, $y = 4 + \sqrt{-x}$ and $y = 4 - \sqrt{-x}$. We choose the latter, since we want $y < 4$.
Answer:

$$y = 4 - \sqrt{-x}$$

4. A box with an open top is to be constructed from a rectangular piece of cardboard with dimensions $b = 12$ in. by $a = 24$ in. by cutting out equal squares of side x at each corner and then folding up the sides as in the figure. Express the volume V of the box as a function of x.
Solution Volume = Height $\times$ Width $\times$ Length $= x(a-2x)(b-2x) = x(24-2x)(12-2x) = 4x(12-x)(6-x) = 4x(x^2-18x+72) = 4x^3 - 72x^2 + 288x$

5. If the point $(6,3)$ is on the graph of an even function, what other point must be on the graph?
Solution Reminder – if a function $f(x)$ satisfies $f(x) = f(-x)$ for all

x, we call $f(x)$ an even function. Now suppose $(6,3)$ is on the graph of $y = f(x)$. This means $3 = f(6)$.So using the fact that $f(x)$ is even we get the following.

$$3 = f(6) = f(-6)$$

Which means that $(-6,3)$ is on the graph.

6. Biologists have noticed that the chirping rate of crickets of certain species is related to temperature, and the relationship appears to be very nearly linear. A cricket produces 113 chirps per minute at 70°F and 173 chirps per minute at 80°F per minute. Find the slope of this line and the rate of chirping, if the temparature is 70°F
Solution Use the slope formula $\frac{y_2-y_1}{x_2-x_1}$, where y's are for chirps/min and x's are for degrees.
$$\frac{173-113}{80-70} = 6.$$
Now for the rate of chirping at 70°F, it is already given in the problem (a mistake?), which is 113 chirps per minute.

7. **Solution** From the original graph $y = f(x)$, other graphs are constructed by applying basic transformations. $y = \frac{1}{2}f(x)$ is contraction in the y direction by half, which is graph 1. $y = 2f(x+2)$ is expanding twice the size in the y direction and shifting 2 to the left, which is graph 3. $y = -f(x+2)$ is reflection over x-axis and shifting 2 to the left, which is graph 2.

8. The graph on the right is expanded in the y direction by 2 and shifted 5 to the right. Applying these tranformations you get $g(x) = 2\sqrt{6(x-5)-(x-5)^2} = 2\sqrt{6x-30-x^2+10x-25} = 2\sqrt{-55-16x-x^2}$.

9. The new graph is obtained by shifting down by 4. So the answer is $g(x) = \cos(x) - 4$.

10. $f(0) = \sin|2 \cdot 0| = \sin 0 = 0$, so the graph must pass the origin. This rules out Graph 1. Looking at Graph 2 and 3, $x = \frac{3}{4}\pi$ seems to be a good choice for distinguishing the two.
$$f(\frac{3}{4}\pi) = \sin|2 \cdot \frac{3}{4}\pi| = \sin\frac{3}{2}\pi = -1$$
. So Graph 2 is the correct one.
Remark The straight forward way to draw the graph of $y = f(x) = \sin|2x|$ is first draw $y = \sin 2x$ for $x > 0$ and then reflecting in over y axis.

11. Consider the functions $f(x) = \sin x$ and $g(x) = 3 - \sqrt{x}$.
    (a) $f \circ g(x) = f\{g(x)\} = f(3-\sqrt{x}) = \sin 3 - \sqrt{x}$
    (b) $g \circ f(x) = g\{f(x)\} = 3 - \sqrt{\sin x}$
    (c) $f \circ f(x) = f\{f(x)\} = f(\sin x) = \sin(\sin x)$
    (d) $g \circ g(x) = g\{g(x)\} = g(3-\sqrt{x}) = 3 - \sqrt{3-\sqrt{x}}$