Answers

1) Your favorite dog groomer charges according to your dog's weight. If your dog is 15 pounds and under, the groomer charges $35. If your dog is between 15 and 40 pounds, she charges $40. If your dog is over 40 pounds, she charges $40, plus an additional $2 for each pound.

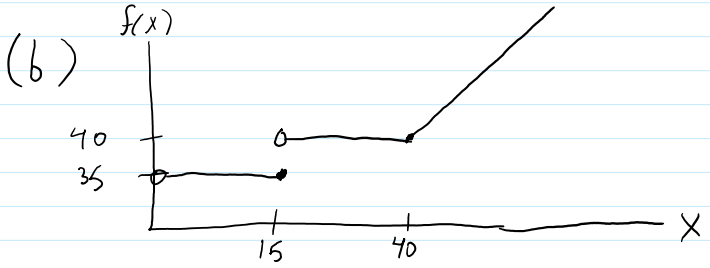
(a) Write a piecewise function that describes what your dog groomer charges.
(b) Graph the function.
(c) What would the groomer charge if your cute dog weighs 60 pounds?

Let $f(x)$ denote the function

(a) $$f(x) = \begin{cases} \$35 & , \ 0 < x \leq 15 \\ \$40 & , \ 15 < x \leq 40 \\ \$(40 + 2(x-40)) & , \ x > 40 \end{cases}$$

$x :=$ Pounds

(b)



Something like this

(c) $f(60) = \$(40 + 2(60-40)) = \$(40 + 40) = \$80$

$\Rightarrow$ **$f(60) = \$80$**

2) What value of ***a*** would make this piecewise function **continuous**?

$$f(x) = \begin{cases} 3x^2 + 4 & \text{if } x < -2 \\ 5x + a & \text{if } x \geq -2 \end{cases}$$

We need to find a value "a" such that at $x = -2$,

$$5x + a = 3x^2 + 4 \quad @ \ x = -2$$

Thus,

$$5(-2) + a = (3)(-2)^2 + 4$$

$$\Rightarrow -10 + a = 12 + 4$$

$$\Rightarrow a = 26$$

Evaluate each piecewise function for $x = -3$ and $x = 4$.

3. $f(x) = \begin{cases} 10 & \text{if } x < 4 \\ -1 & \text{if } x \geq 4 \end{cases}$

4. $g(x) = \begin{cases} 3x - 1 & \text{if } x < 0 \\ 2x^2 & \text{if } 0 \leq x < 4 \\ 1 - x & \text{if } 4 \leq x \end{cases}$

5. $h(x) = \begin{cases} 2 - x & \text{if } x < -2 \\ 4x & \text{if } -2 \leq x < 3 \\ 6 - x & \text{if } 3 \leq x \end{cases}$

6. $k(x) = \begin{cases} -12 & \text{if } x < 1 \\ 7x & \text{if } 1 \leq x \leq 4 \\ 2x - x^2 & \text{if } 4 < x \end{cases}$

3) $f(-3) = 10$, $f(4) = -1$

4) $g(-3) = 3(-3) - 1 = -10 \Rightarrow g(-3) = -10$

$g(4) = 1 - 4 = -3 \Rightarrow g(4) = -3$

5) $h(-3) = 2 - (-3) = 5 \Rightarrow h(-3) = 5$

$h(4) = 6 - 4 = 2 \Rightarrow h(4) = 2$

6) $k(-3) = -12$

$k(4) = 7(4) = 28$

$\Rightarrow k(-3) = -12$, $k(4) = 28$

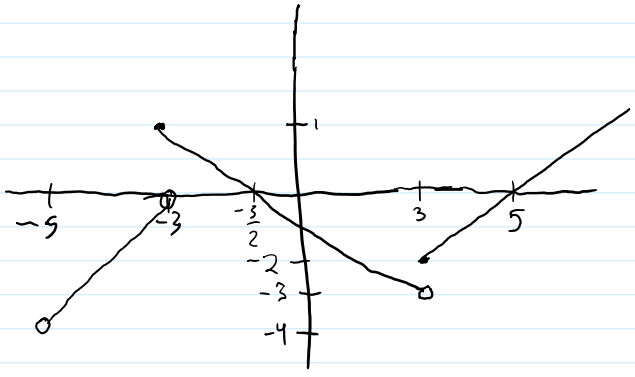
7) Graph the following piecewise functions:

a) $f(x) = \begin{cases} 2x + 6 & \text{if } -5 < x < -3 \\ -\frac{2}{3}x - 1 & \text{if } -3 \leq x < 3 \\ x - 5 & \text{if } x \geq 3 \end{cases}$

b) $f(x) = \begin{cases} \frac{1}{2}x + 5 & \text{if } x \leq -2 \\ -\frac{2}{3}x - 2 & \text{if } -2 < x < 0 \\ 4x - 4 & \text{if } x \geq 0 \end{cases}$

c) $f(x) = \begin{cases} x^2 - 2x + 1 & \text{if } -1 \leq x < 2 \\ \frac{1}{2}x + 1 & \text{if } x \geq 2 \end{cases}$

a)

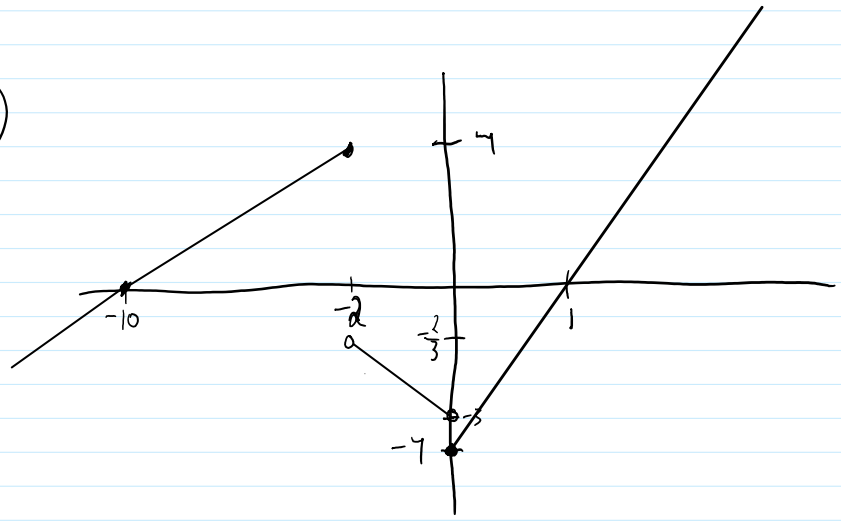


$\begin{cases} 2x + 6 = 0 \\ \Rightarrow x = -3 \end{cases}$ $2(-5) + 6 = -4$

$\begin{cases} -\frac{2}{3}x - 1 = 0 \\ \Rightarrow x = -\frac{3}{2} \\ -\frac{2}{3}(-3) - 1 = 1 \\ -\frac{2}{3}(3) - 1 = -3 \end{cases}$

$\begin{cases} x - 5 = 0 \Rightarrow x = 5 \\ 3 - 5 = -2 \end{cases}$

b)

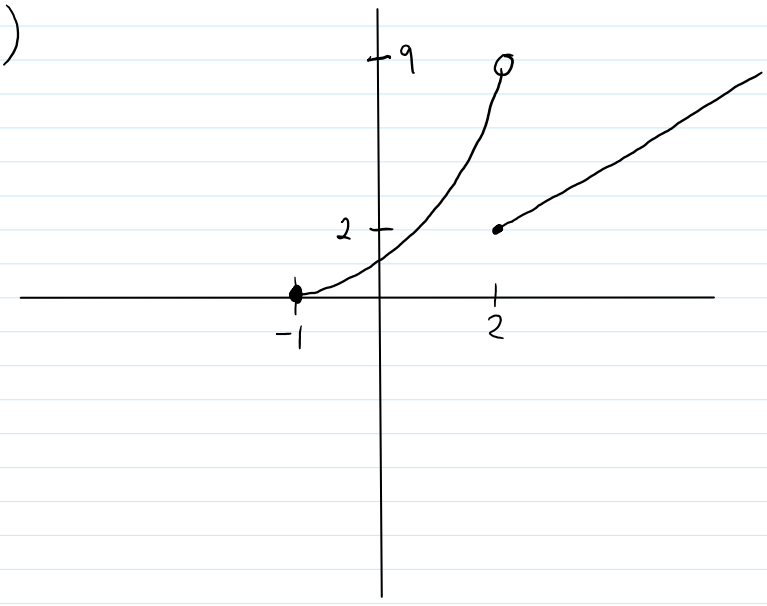


$\begin{cases} \frac{1}{2}x+5=0 \\ \Rightarrow x=-10 \\ \frac{1}{2}(-2)+5=4 \end{cases}$

$\begin{cases} -\frac{2}{3}x-2=0 \\ \Rightarrow x=-3 \\ -\frac{2}{3}(-2)-2=\frac{4}{3}-2=-\frac{2}{3} \end{cases}$

$\begin{cases} 4x-4=0 \\ \Rightarrow x=1 \\ 4(0)-4=-4 \end{cases}$

c)



$\begin{cases} x^2+2x+1=0 \\ \Rightarrow (x+1)^2=0 \\ \Rightarrow x=-1 \\ f(2)=(2+1)^2=9 \end{cases}$ $x\in[-1,2]$

$\begin{cases} \frac{1}{2}x+1=0 \\ x=-2 \quad \text{(not in interval)} \\ f(2)=2 \end{cases}$ $x\geq 2$

8) The rate of decay of a certain sub-atomic particle at a temperature of 0° C and lower is modeled by the equation $f(x)=x^2+4x$. At temperatures above 0° C its rate of decay is modeled by the equation $f(x)=-x^2+4x+2$. Please model the rate of decay with a piecewise function and graph below.

$$f(x)=\begin{cases} x^2+4x, & x\leq 0 \\ -x^2+4x+2, & x>0 \end{cases}$$

Notice that

Notice that

$$x^2+4x = x^2+4x+\left(\frac{4}{2}\right)^2-\left(\frac{4}{2}\right)^2$$

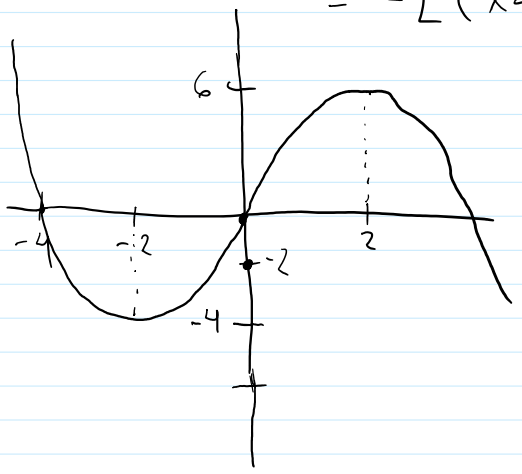
$$= (x^2+4x+4)-4 = \boxed{(x+2)^2-4}$$

$$-x^2+4x+2 = -(x^2-4x-2)$$

$$= -[(x^2-4x+4)-4-2]$$

$$= -[(x-2)^2-6] = -(x-2)^2+6$$

Thus,



$(x+2)^2-4=0$

$x+2=\pm 2$

$x=0,\ x=-4$

$(x-2)^2-6=0$

$x=2\pm\sqrt{6}$