MATH 123 Solutions to Exam 2, Larry version

2 points 1. Represent $\log_3(27)$ as an integer, fraction, or radical.

Solution: Since $3^3 = 27$, $\log_3(27) = 3$.

1. 3

2 points 2. What is the largest domain on which the function $\ln(2x+1)$ is defined?

Solution: Since $\ln y$ is defined for $y > 0$, we must have $2x + 1 > 0$. Thus $2x > -1$, so $x > -1/2$.

2. $x > -\frac{1}{2}$

2 points 3. Find x if $8^{4x+1} = 64$.

Solution: Since $8^{4x+1} = 64$ and $8^2 = 64$, we solve $4x + 1 = 2$. Thus $4x = 1$ so $x = 1/4$.

3. $\frac{1}{4}$

2 points 4. Write the equation of the line that passes through the points $(1, 2e)$ and $(e, 2)$.

Solution: The slope of the line is $\frac{2e-2}{1-e} = -2$. Thus, the equation is $y - 2 = -2(x - e)$.

4. $y = 2 - 2(x - e)$

2 points 5. If $\log x = 4$ and $\log y = 6$, simplify $\log\left(\frac{x^2}{\sqrt[3]{y}}\right)$ as much as possible.

Solution: $\log\left(\frac{x^2}{\sqrt[3]{y}}\right) = 2\log x - \frac{1}{3}\log y = 8 - 2 = 6$.

5. 6

2 points 6. Write the equation of a circle with center at $(5, -2)$ and radius 6.

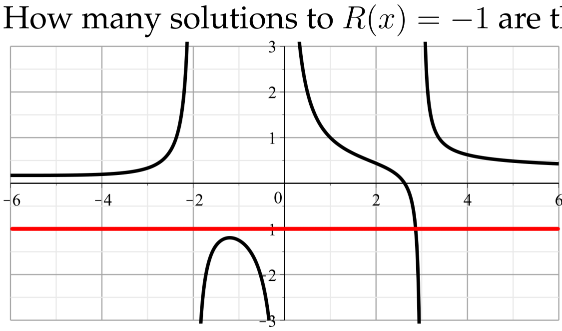
6. $(x-5)^2 + (y+2)^2 = 36$

2 points 7. Represent $\log_8(32)$ as an integer, fraction, or radical.

Solution: We must find y so that $8^y = 32$. Since $8 = 2^3$ and $32 = 2^5$, we need y so that $(2^3)^y = 2^{3y} = 2^5$, or $3y = 5$. Hence $y = \frac{5}{3}$.

7. $\frac{5}{3}$

2 points 8. Let $R(x)$ with domain $[-6, 6]$ have the graph below.
How many solutions to $R(x) = -1$ are there?



8. **1**

8 points 9. Using polynomial division, write the following quotient as a polynomial plus a remainder term where the numerator has degree less than the denominator (there may be no remainder).

$$\frac{x^4 + 2x^3 + x^2 + 37x + 4}{x + 4}$$

Solution:

$$\begin{array}{r|l} & x^3 - 2x^2 + 9x + 1 \\ \hline x+4 & x^4 + 2x^3 + x^2 + 37x + 4 \\ & -x^4 - 4x^3 \\ \hline & -2x^3 + x^2 \\ & 2x^3 + 8x^2 \\ \hline & 9x^2 + 37x \\ & -9x^2 - 36x \\ \hline & x + 4 \\ & -x - 4 \\ \hline & 0 \end{array}$$

So the result is

$$x^3 - 2x^2 + 9x + 1$$

and there is no remainder.

8 points 10. Find all real values of x that solve the equation below. Give an **exact** answer– that is, do not try to approximate logarithms, powers of e, $\sqrt{2}$, etc.

$$2^{3x-1} = 5^x$$

Solution: Take the logarithm of both sides to get

$$\begin{aligned} \ln\left(2^{3x-1}\right) &= \ln\left(5^x\right) \\ (3x-1)\ln 2 &= x \ln 5 \\ 3x - 1 &= x\frac{\ln 5}{\ln 2} \\ 3x - x\frac{\ln 5}{\ln 2} &= 1 \\ x\left(3 - \frac{\ln 5}{\ln 2}\right) &= 1 \\ x &= \frac{1}{3 - \frac{\ln 5}{\ln 2}} = \frac{\ln 2}{3\ln 2 - \ln 5} \end{aligned}$$

11. In your secret laboratory in the tunnels near Harriman, you are brewing a batch of goop (which is delicious on udon noodles!). The growth of goop is fueled by bacteria, and so it grows exponentially. You start your batch with 5 kg. of goop, and after 6 hours you have 12 kg.

4 points (a) Give an equation for $G(t)$, the amount of goop (in kg.) that you will have t hours after you begin your brew.

Solution: Since the goop grows exponentially, $G(t) = Ae^{rt}$, and we need to figure out the constants A and r. Since we started with 5 kg. $G(0) = 5 = Ae^0$, so $A = 5$. Now we also know that when $t = 6$, we have 12 kg, so $G(6) = 12 = 5e^{6r}$. Solving for r, we have $\frac{12}{5} = e^{6r}$; taking the log of both sides gives $\ln \frac{12}{5} = 6r$, or $r = \ln\left(\frac{12}{5}\right)/6$. This means our answer is

$$G(t) = 5e^{t\ln\left(\frac{12}{5}\right)/6} = 5\left(\frac{12}{5}\right)^{t/6}$$

4 points (b) You will need 100 kg. of goop within one day of when you started. Will you have enough? You must justify your answer for full credit.

Solution: Since a day is 24 hrs long, we just want to know if $G(24) = 5\left(\frac{12}{5}\right)^{24/6}$ is bigger than 100, or equivalently, whether $\left(\frac{12}{5}\right)^4 \geq 20$. It is. (In fact, there's more than 165 kg. of goop after 24 hours.)

12. Answer the questions below for the function $Q(r) = \dfrac{5r^2 - 25}{r^2 + 2r - 3}$.

2 points (a) What is the largest domain on which $Q(r)$ is defined?

Solution: The numerator is defined for all values of r, so we only have to see where the denominator is zero (or undefined). Factor $r^2 + 2r - 3$ as $(r+3)(r-1)$ and see that $r \neq -3$ and $r \neq 1$.

(a) **All r such that $r \neq -3$ and $r \neq 1$.**

2 points (b) What value (if any) does $Q(r)$ approach as r approaches infinity? (If there is none, write "DNE")

Solution: For r very large, $\dfrac{5r^2 - 25}{r^2 + 2r - 3} \approx \dfrac{5r^2}{r^2} = 5$

(b) **5**

2 points (c) What, if any, are the zeroes of $Q(r)$? (if there are none, write "none")

Solution: We will have a zero if the numerator is zero. $5r^2 - 25 = 5(r^2 - 5) = 5(r - \sqrt{5})(r + \sqrt{5})$

(c) $r = \pm\sqrt{5}$

2 points (d) At what y value (if any) does the graph of $y = Q(r)$ cross the y-axis? (if there is no such y, write "none")

Solution: We just need to evaluate $Q(0)$.

$Q(0) = \dfrac{0 - 25}{0 + 0 - 3} = \dfrac{25}{3}$.

(d) $\dfrac{25}{3}$

8 points 13. Your friend gives you a cache of the radioactive element unstablium, which you take it back to your secret lab in the tunnels. Unstablium decays at an exponential rate into boringite. Initally, you have 80 grams of unstablium. Three hours later, you have 60 grams. What is the half-life of unstablium?

Solution: Let $U(t)$ denote the amount of unstablium we have (in grams) after t hours. Since we start with 80 grams, we have $U(t) = 80e^{rt}$, and we need to figure out r. Since there are 60 grams after 3 hours, we know that $60 = U(3) = 80e^{-3r}$, or $\dfrac{3}{4} = e^{-3r}$. Taking the log of both sides, we get $\ln(3/4) = 3r$, so $r = \ln(3/4)/3$.

$$U(t) = 80e^{t\ln(3/4)/3} = 80\left(\frac{3}{4}\right)^{t/3}$$

The half life is the time it takes for half of the material to be left. So we need to solve $U(t) = 40$ for t.

$$40 = 80\left(\frac{3}{4}\right)^{t/3}$$

$$\frac{1}{2} = \left(\frac{3}{4}\right)^{t/3}$$

$$\ln\left(\frac{1}{2}\right) = \frac{t}{3}\ln\left(\frac{3}{4}\right)$$

$$\frac{\ln\left(\frac{1}{2}\right)}{\ln\left(\frac{3}{4}\right)} = \frac{t}{3}$$

$$3\frac{\ln\left(\frac{1}{2}\right)}{\ln\left(\frac{3}{4}\right)} = t$$

This can be rewritten (using log rules) as $\dfrac{\ln 8}{\ln 4 - \ln 3}$, but you don't have to. (This is about 7.2 hours).