

(1a)

$$\begin{array}{r}
 x^4 + 6x^3 - 3x^2 + 5x + 7 \\
 x-2 \overline{) x^5 + 4x^4 - 15x^3 + 11x^2 - 3x - 14} \\
 \underline{-(x^5 - 2x^4)} \\
 6x^4 - 15x^3 \\
 \underline{-(6x^4 - 12x^3)} \\
 -3x^3 + 11x^2 \\
 \underline{-(-3x^3 + 6x^2)} \\
 5x^2 - 3x \\
 \underline{-(5x^2 - 10x)} \\
 7x - 14 \\
 \underline{-(7x - 14)} \\
 0
 \end{array}$$

(b)

$$\begin{array}{r}
 x^3 + 9x^2 - 11x + 10 \\
 x+4 \overline{) x^4 + 13x^3 + 25x^2 - 34x + 10} \\
 \underline{-(x^4 + 4x^3)} \\
 9x^3 + 25x^2 \\
 \underline{-(9x^3 + 36x^2)} \\
 -11x^2 - 34x \\
 \underline{-(-11x^2 - 44x)} \\
 10x + 40 \\
 \underline{-(10x + 40)} \\
 0
 \end{array}$$

1c

$$\begin{array}{r} x^4 - 6x^3 + 9x^2 - 8x + 2 \\ x+8 \overline{) x^5 + 2x^4 - 39x^3 + 64x^2 - 62x + 16} \\ \underline{-(x^5 + 8x^4)} \\ -6x^4 - 39x^3 \\ \underline{-(-6x^4 - 48x^3)} \\ 9x^3 + 64x^2 \\ \underline{-(9x^3 + 72x^2)} \\ -8x^2 - 62x \\ \underline{-(-8x^2 - 64x)} \\ 2x + 16 \\ \underline{2x + 16} \\ 0 \end{array}$$

d

$$\begin{array}{r} x^3 - 7x^2 - 6x + 10 \\ x+5 \overline{) x^4 - 2x^3 - 41x^2 - 20x + 50} \\ \underline{-(x^4 + 5x^3)} \\ -7x^3 - 41x^2 \\ \underline{-(-7x^3 - 35x^2)} \\ -6x^2 - 20x \\ \underline{-(-6x^2 - 30x)} \\ 10x + 50 \\ \underline{-(10x + 50)} \\ 0 \end{array}$$

$$(a) e^{2x} - 4e^x - 5 = 0 \quad \text{let } y = e^x$$

$$y^2 - 4y - 5 = 0$$

$$(y-5)(y+1) = 0$$

$$y = 5, -1 \rightarrow e^x = 5, e^x = -1$$

$$\boxed{x = \ln 5}$$

$$(b) e^{2x} + 4e^x - 12 = 0 \quad \text{let } y = e^x$$

$$y^2 + 4y - 12 = 0$$

$$(y+6)(y-2) = 0$$

$$y = -6, 2 \rightarrow e^x = -6, e^x = 2$$

$$\boxed{x = \ln 2}$$

$$(c) \left(e^x + \frac{24}{e^x} = 11 \right) e^x$$

$$e^{2x} + 24 = 11e^x$$

$$e^{2x} - 11e^x + 24 = 0 \quad \text{let } y = e^x$$

$$y^2 - 11y + 24 = 0$$

$$(y-3)(y-8) = 0$$

$$y = 3, 8 \rightarrow e^x = 3, e^x = 8$$

$$\boxed{x = \ln 3, x = \ln 8}$$

$$(d) \left(2e^x - \frac{2}{e^x} = -3 \right) e^x$$

$$2e^{2x} - 2 = -3e^x$$

$$2e^{2x} + 3e^x - 2 = 0 \quad \text{let } y = e^x$$

$$2y^2 + 3y - 2 = 0$$

$$(2y-1)(y+2) = 0$$

$$y = \frac{1}{2}, y = -2 \rightarrow e^x = \frac{1}{2}, e^x = -2$$

$$\boxed{x = \ln \frac{1}{2}}$$

$$\textcircled{20} \quad e^{2x} + 8e^x + 15 = 0 \quad \text{let } y = e^x$$

$$y^2 + 8y + 15 = 0$$

$$(y+3)(y+5) = 0$$

$$y = -3, -5 \rightarrow e^x = -3, e^x = -5$$

No solutions

$$\#3) \quad 9^{x-4} = 10^x$$

$$\log 9^{x-4} = \log 10^x$$

$$(x-4)\log 9 = x$$

$$x \log 9 - 4 \log 9 = x$$

$$x \log 9 - x = 4 \log 9$$

$$x(\log 9 - 1) = 4 \log 9$$

$$x = \frac{4 \log 9}{\log 9 - 1}$$



NOTE THAT IN THIS SOLUTION, \log MEANS THE BASE 10 LOGARITHM. IF YOU USE \ln , YOU'D GET

$$\frac{4 \ln 9}{\ln 9 - \ln 10}$$

$$4^{1-6x} = 8^{3x+2}$$

$$(2^2)^{1-6x} = (2^3)^{3x+2}$$

$$2^{2-12x} = 2^{9x+6}$$

$$2-12x = 9x+6$$

$$-4 = 21x$$

$$x = \frac{-4}{21}$$

$$\#3) 7^{6-4x} = e^x$$

$$\ln 7^{6-4x} = \ln e^x$$

$$(6-4x)\ln 7 = x$$

$$6\ln 7 - 4x\ln 7 = x$$

$$6\ln 7 = x + 4x\ln 7$$

$$6\ln 7 = x(1 + 4\ln 7)$$

$$\boxed{\frac{6\ln 7}{1 + 4\ln 7} = x}$$

$$5^{x+1} = 25^{3-3x}$$

$$5^{x+1} = (5^2)^{3-3x}$$

$$5^{x+1} = 5^{6-6x}$$

$$x+1 = 6-6x$$

$$7x = 5$$

$$\boxed{x = \frac{5}{7}}$$

$$2^{5+3x} = 3^x$$

$$\log 2^{5+3x} = \log 3^x$$

$$(5+3x)\log 2 = x\log 3$$

$$5\log 2 + 3x\log 2 = x\log 3$$

$$5\log 2 = x\log 3 - 3x\log 2$$

$$5\log 2 = x(\log 3 - 3\log 2)$$

$$\boxed{\frac{5\log 2}{\log 3 - 3\log 2} = x}$$

$$(4a) (i) g(8) = \frac{16}{8} = 2 \rightarrow f'(2) = \frac{1}{4}(2^4) - 4(2) - 2 = -6$$

$$(ii) f(g(x)) = \frac{1}{4} \left(\frac{16}{x} \right)^2 - 4 \left(\frac{16}{x} \right) - 2$$

$$(iii) g(f(x)) = \frac{16}{\frac{1}{4}x^2 - 4x - 2}$$

$$(iv) y = \frac{16}{x} \rightarrow x = \frac{16}{y} \rightarrow xy = 16 \rightarrow y = \frac{16}{x} = g^{-1}(x)$$

$$(b) (i) f(4) = \frac{4^3 - 8}{7} = \frac{56}{7} = 8 \quad \cancel{g(f(4)) = 3(4) - 2 = 10} \quad \text{DOH!} \quad g(f(4)) = g(8) = 3 \cdot 8 - 2 = 22$$

$$(ii) f(g(x)) = \frac{(3x-2)^3 - 8}{7}$$

$$(iii) g(f(x)) = 3 \left(\frac{x^3 - 8}{7} \right) - 2$$

$$(iv) y = \frac{x^3 - 8}{7} \rightarrow x = \sqrt[3]{7y + 8} \rightarrow 7x + 8 = y^3 \rightarrow y = \sqrt[3]{7x + 8} \quad f^{-1}(x) = \sqrt[3]{7x + 8}$$

$$(c) (i) g(13) = \sqrt[3]{2(13)+1} = \sqrt[3]{27} = 3 \rightarrow f(3) = \sqrt{2(3)-4} = \sqrt{2}$$

$$(ii) f(g(x)) = \sqrt{2(\sqrt[3]{2x+1}) - 4}$$

$$(iii) g(f(x)) = \sqrt[3]{2(\sqrt{2x-4}) + 1}$$

$$(iv) y = \sqrt[3]{2x+1} \rightarrow x = \frac{y^3 - 1}{2} \rightarrow \frac{y^3 - 1}{2} = x \rightarrow \frac{y^3}{2} = x + \frac{1}{2} \rightarrow \frac{y^3}{2} = x + \frac{1}{2} \rightarrow y = \sqrt[3]{2x + 1}$$

$$g^{-1}(x) = \frac{x^3}{2}$$

(4d) (i) $f(-1) = (-1)^2 + 6(-1) = -5 \rightarrow f(-5) = (-5)^2 - 5 = -130$

(ii) $f(g(x)) = (x^2 + 6x)^3 - 5$

(iii) $g(f(x)) = (x^3 - 5)^2 + 6(x^3 - 5)$

(iv) $y = x^3 - 5 \rightarrow x = y^3 - 5 \rightarrow \sqrt[3]{x+5} = y \mid f^{-1}(x) = \sqrt[3]{x+5}$

(5a) $\sin 2x + \sin x = 0$
 $2\sin x \cos x + \sin x = 0$

$\sin x (2\cos x + 1) = 0$
 $\sin x = 0 \quad 2\cos x + 1 = 0 \rightarrow \cos x = -\frac{1}{2}$

$x = 0, \pi$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

(5b) $4\sin^2 x - 3 = 0$

$\sin^2 x = \frac{3}{4}$

$\sin x = \pm \frac{\sqrt{3}}{2}$

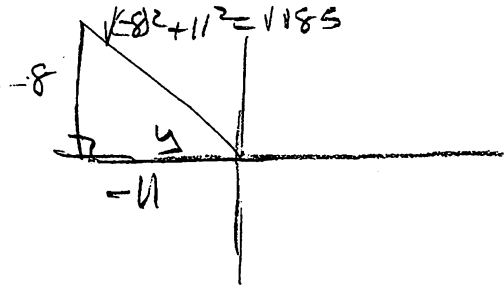
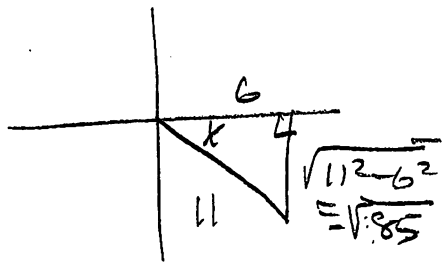
$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

(c) $\cos^2 x - 3\cos x + 2 = 0$

$(\cos x - 2)(\cos x - 1) = 0$

~~$\cos x = 2$~~ $\cos x = 1$
 $x = 0$

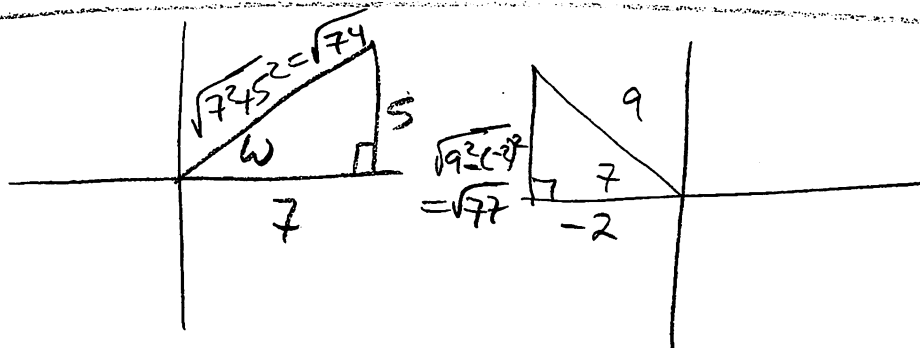
(6b)



$$\begin{aligned} \sin(x+y) &= \sin x \cos y + \cos x \sin y \\ &= \left(\frac{-\sqrt{85}}{11}\right)\left(\frac{-11}{\sqrt{185}}\right) + \left(\frac{6}{11}\right)\left(\frac{8}{\sqrt{185}}\right) = \boxed{\frac{11\sqrt{85} + 48}{11\sqrt{185}}} \end{aligned}$$

$$\cos 2y = \cos^2 y - \sin^2 y = \left(\frac{-11}{\sqrt{185}}\right)^2 - \left(\frac{8}{\sqrt{185}}\right)^2 = \frac{121 - 64}{185} = \boxed{\frac{57}{185}}$$

(c)



$$\begin{aligned} \cos(w+z) &= \cos w \cos z - \sin w \sin z \\ &= \left(\frac{7}{\sqrt{74}}\right)\left(\frac{-2}{9}\right) - \left(\frac{5}{\sqrt{74}}\right)\left(\frac{\sqrt{77}}{9}\right) = \boxed{\frac{-14 - 5\sqrt{77}}{9\sqrt{74}}} \end{aligned}$$

$$\sin 2w = 2 \sin w \cos w = 2\left(\frac{5}{\sqrt{74}}\right)\left(\frac{7}{\sqrt{74}}\right) = \boxed{\frac{70}{74}}$$

$$\cos 2w = \cos^2 w - \sin^2 w = \left(\frac{7}{\sqrt{74}}\right)^2 - \left(\frac{5}{\sqrt{74}}\right)^2 = \frac{49 - 25}{74} = \boxed{\frac{24}{74}}$$

(7a) $f(x) = \frac{5x^2 - 25x - 30}{x^2 + 3x - 28} = \frac{5(x^2 - 5x - 6)}{x^2 + 3x - 28} = \frac{5(x-6)(x+1)}{(x+7)(x-4)}$

a) Domain: $x \neq 4, -7$

b) as $x \rightarrow \infty$, $f(x) \rightarrow 5$

c) $x = 6, x = -1$

d) $\frac{5(0-6)(0+1)}{(0+7)(0-4)} = \frac{-30}{-28} = \frac{30}{28}$

b) $f(x) = \frac{x^2 - 16}{4(x^2 - x - 6)} = \frac{(x-4)(x+4)}{4(x-3)(x+2)}$

a) Domain: $x \neq 3, -2$

b) as $x \rightarrow \infty$, $f(x) \rightarrow \frac{1}{4}$

c) $x = 4, x = -4$

d) $\frac{(0-4)(0+4)}{4(0-3)(0+2)} = \frac{-16}{-24} = \frac{16}{24}$

c) $f(x) = \frac{2(x^2 - x - 72)}{3(x^2 - 6x - 40)} = \frac{2(x-9)(x+8)}{3(x-10)(x+4)}$

a) Domain: $x \neq 10, -4$

b) as $x \rightarrow \infty$, $f(x) \rightarrow \frac{2}{3}$

c) $x = 9, x = -8$

d) $\frac{2(0-9)(0+8)}{3(0-10)(0+4)} = \frac{-144}{-120} = \frac{144}{120}$

(Pa)

$$y = a \cdot b^x$$

$$(0, 800) \quad (10, 650)$$

$$800 = a \cdot b^0$$

$$800 = a$$

$$y = 800 b^x$$

$$650 = 800 b^{10}$$

$$\frac{13}{16} = b^{10}$$

$$\left(\frac{13}{16}\right)^{\frac{1}{10}} = b$$

$$y = 800 \left(\frac{13}{16}\right)^{\frac{x}{10}}$$

$$400 = 800 \left(\frac{13}{16}\right)^{\frac{x}{10}}$$

$$\frac{1}{2} = \left(\frac{13}{16}\right)^{\frac{x}{10}}$$

$$\log\left(\frac{1}{2}\right) = \log\left(\frac{13}{16}\right)^{\frac{x}{10}} = \frac{x}{10} \log\left(\frac{13}{16}\right)$$

$$\boxed{\frac{10 \log\left(\frac{1}{2}\right)}{\log\left(\frac{13}{16}\right)} = x}$$

b) $y = a \cdot b^x$

$$(0, 12) \quad (2, 1)$$

$$12 = a \cdot b^0$$

$$a = 12$$

$$y = 12 \cdot b^x$$

$$1 = 12 \cdot b^2$$

$$\frac{1}{12} = b^2$$

$$b = \left(\frac{1}{12}\right)^{\frac{1}{2}}$$

$$y = 12 \left(\frac{1}{12}\right)^{\frac{x}{2}}$$

$$6 = 12 \left(\frac{1}{12}\right)^{\frac{x}{2}}$$

$$\frac{1}{2} = \left(\frac{1}{12}\right)^{\frac{x}{2}}$$

$$\log\left(\frac{1}{2}\right) = \log\left(\frac{1}{12}\right)^{\frac{x}{2}} = \frac{x}{2} \log\left(\frac{1}{12}\right)$$

$$\boxed{\frac{2 \log\left(\frac{1}{2}\right)}{\log\left(\frac{1}{12}\right)} = x}$$

(8c) $y = a \cdot b^x$ (0, 70) (10, 10)

$$70 = a \cdot b^0$$

$$a = 70$$

$$y = 70 b^x$$

$$10 = 70 b^{10}$$

$$\frac{1}{7} = b^{10}$$

$$b = \left(\frac{1}{7}\right)^{\frac{1}{10}}$$

$$y = 70 \left(\frac{1}{7}\right)^{\frac{x}{10}}$$

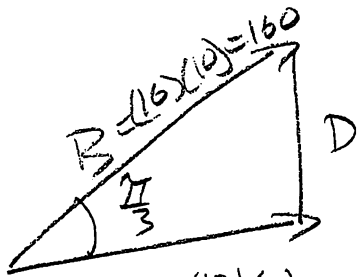
$$35 = 70 \left(\frac{1}{7}\right)^{\frac{x}{10}}$$

$$\frac{1}{2} = \left(\frac{1}{7}\right)^{\frac{x}{10}}$$

$$\log \frac{1}{2} = \log \left(\frac{1}{7}\right)^{\frac{x}{10}} = \frac{x}{10} \log \left(\frac{1}{7}\right)$$

$$\frac{10 \log \left(\frac{1}{2}\right)}{\log \left(\frac{1}{7}\right)} = x$$

(9a)



$$A = (12)(10) = 120$$

$$D^2 = (120)^2 + (160)^2 - 2(120)(160) \cos \frac{2\pi}{3}$$

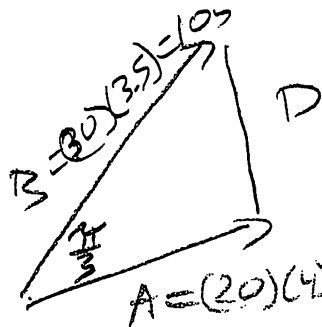
$$D^2 = 14400 + 25600 - 38400 \left(\frac{1}{2}\right)$$

$$D^2 = 40000 - 19200$$

$$D^2 = 20800$$

$$D = \sqrt{20800}$$

(ab)



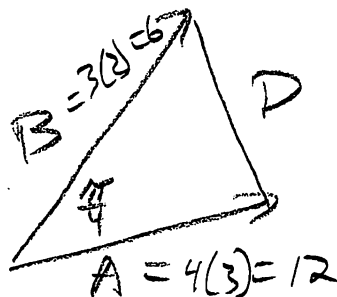
$$D^2 = 80^2 + 105^2 - 2(80)(105) \cos \frac{\pi}{3}$$

$$D^2 = 6400 + 11,025 - 16,800 \left(\frac{1}{2}\right)$$

$$D^2 = 17,425 - 8400 = 9025$$

$$D = \sqrt{9025}$$

c)



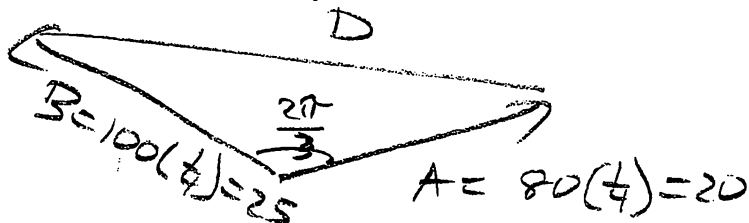
$$D^2 = 12^2 + 6^2 - 2(12)(6) \cos \frac{\pi}{4}$$

$$D^2 = 144 + 36 - 144 \left(\frac{\sqrt{2}}{2}\right)$$

$$D^2 = 180 - 72\sqrt{2}$$

$$D = \sqrt{180 - 72\sqrt{2}}$$

d)



$$D^2 = 20^2 + 25^2 - 2(20)(25) \cos \left(\frac{2\pi}{3}\right)$$

$$D^2 = 400 + 625 - 1000 \left(-\frac{1}{2}\right)$$

$$D^2 = 1025 + 2000 = 3025$$

$$D = \sqrt{3025}$$