① $y=\dfrac{x^2-7x+12}{x^2-1}=\dfrac{(x-3)(x-4)}{(x+1)(x-1)}$ ①

a) $x=3,\ x=4$

b) $y=-12$

c) Domain: $x\neq\pm1$

d) $y=1$

$y=\dfrac{2x^3-6x^2}{x^2-4x+3}=\dfrac{2x^2\cancel{(x-3)}}{(x-1)\cancel{(x-3)}}$

a) $x=0$

b) $y=0$

c) Domain: $x\neq 1,3$

d) DNE

$y=\dfrac{3x-5}{x^2-8x+7}=\dfrac{3x-5}{(x-7)(x-1)}$

a) $x=\frac{5}{3}$

b) $y=-\frac{5}{7}$

c) Domain: $x\neq 1,7$

d) $y=0$

$y=\dfrac{4x^2-8x-32}{x^2+4}=\dfrac{4(x^2-2x-8)}{x^2+4}=\dfrac{4(x-4)(x+2)}{x^2+4}$

a) $x=4,\ x=-2$

b) $y=-8$

c) All reals

d) $y=4$

2)

$$x^3-4x^2+2x+1$$

$$x-3 \overline{)\, x^4-7x^3+14x^2-5x-3}$$

$$-(x^4-3x^3)$$

$$-4x^3+14x^2$$

$$-(-4x^3+12x^2)$$

$$2x^2-5x$$

$$-(2x^2-6x)$$

$$x-3$$

Quotient is x^3-4x^2+2x+1

---

b)

$$2x^3+5x^2-3x-2$$

$$x+4 \overline{)\, 2x^4+13x^3+17x^2-14x-8}$$

$$-(2x^4+8x^3)$$

$$5x^3+17x^2$$

$$-(5x^3+20x^2)$$

$$-3x^2-14x$$

$$-(-3x^2-12x)$$

$$-2x-8$$

Quotient is $2x^3+5x^2-3x-2$

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c)

$$4x^3-6x^2+3x-5$$

$$2x+5 \overline{)\, 8x^4+8x^3-24x^2+5x-25}$$

$$-(8x^4+20x^3)$$

$$-12x^3-24x^2$$

$$-(-12x^3-30x^2)$$

$$6x^2+5x$$

$$-(6x^2+15x)$$

$$-10x-25$$

Quotient is $4x^3-6x^2+3x-5$

$$
\begin{array}{r}
x^4-8x^3+6x^2-4x+3 \\
x-2\overline{)\,x^5-10x^4+22x^3-16x^2+11x-6} \\
-(x^5-2x^4) \\
\hline
-8x^4+22x^3 \\
-(-8x^4+16x^3) \\
\hline
6x^3-16x^2 \\
-(6x^3-12x^2) \\
\hline
-4x^2+11x \\
-(-4x^2+8x) \\
\hline
3x-6
\end{array}
$$

Quotient is $x^4-8x^3+6x^2-4x+3$

(3) slope is $\frac{2-(-4)}{5-8} = \frac{6}{-3} = -2$

$y-2=-2(x-5)$

slope is $\frac{4-(-5)}{-6-1} = -\frac{9}{7}$

$y-4=\frac{-9}{7}(x+6)$

slope is $\frac{11-7}{8-(-6)} = \frac{4}{14} = \frac{2}{7}$

$y-11=\frac{2}{7}(x-8)$

slope is $\frac{-8+4}{2-8} = \frac{-4}{-6} = \frac{2}{3}$

$y+8=\frac{2}{3}(x-2)$

4) $(x-5)^2+(y+6)^2=7^2$

$(x+4)^2+(y-9)^2=11^2$

$(x+3)^2+(y+10)^2=5^2$

$(x-8)^2+(y+5)^2=9^2$

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5) $4^{x-2}=8^{3-x}$

$(2^2)^{x-2}=(2^3)^{3-x}$

$2^{2x-4}=2^{9-3x}$

$2x-4=9-3x$

$5x=13$

$x=\frac{13}{5}$

$5^{2x+3}=25^{4x+1}$

$5^{2x+3}=(5^2)^{4x+1}$

$5^{2x+3}=5^{8x+2}$

$2x+3=8x+2$

$1=6x$

$x=\frac{1}{6}$

$27^{4-x}=81^{2+5x}$

$(3^3)^{4-x}=(3^4)^{2+5x}$

$3^{12-3x}=3^{8+20x}$

$12-3x=8+20x$

$4=23x$

$x=\frac{4}{23}$

$6^{x+3} = 12$

$\log 6^{x+3} = \log 12$

$(x+3)\log 6 = \log 12$

$x+3 = \dfrac{\log 12}{\log 6}$

$x = \dfrac{\log 12}{\log 6} - 3$ or $\log_6 12 - 3$

$3^{x+5} = 20$

$\log 3^{x+5} = \log 20$

$(x+5)\log 3 = \log 20$

$x+5 = \dfrac{\log 20}{\log 3}$

$x = \dfrac{\log 20}{\log 3} - 5$ or $\log_3 20 - 5$

$2^{3x+1} = 6^x$

$\log 2^{3x+1} = \log 6^x$

$(3x+1)\log 2 = x\log 6$

$3x\log 2 + \log 2 = x\log 6$

$3x\log 2 - x\log 6 = -\log 2$

$x(3\log 2 - \log 6) = -\log 2$

$x = \dfrac{-\log 2}{3\log 2 - \log 6}$

$$\log_3(4x+1)=2$$
$$4x+1=3^2=9$$
$$4x=8$$
$$x=2$$

$$\log_5(57-4x)=3$$
$$57-4x=5^3=125$$
$$-4x=68$$
$$x=-17$$

$$\log_4(2x+1)=-1$$
$$2x+1=4^{-1}=\frac{1}{4}$$
$$2x=-\frac{3}{4}$$
$$x=-\frac{3}{8}$$

$$\ln(x-5)=2$$
$$x-5=e^2$$
$$x=5+e^2$$

$$\ln(3x-2)=4$$
$$3x-2=e^4$$
$$3x=e^4+2$$
$$x=\frac{e^4+2}{3}$$

7 $\log_3(4x-6)-\log_3(x+1)=2$

$\log_3\left(\frac{4x-6}{x+1}\right)=2$

$\frac{4x-6}{x+1}=3^2=9$

$4x-6=9x+9$

$-15=5x$

$x=-3$

You can't take the log of a negative number so no solution

$\log_2(5x+12)-\log_2(x-3)=3$

$\log_2\left(\frac{5x+12}{x-3}\right)=3$

$\frac{5x+12}{x-3}=2^3=8$

$5x+12=8x-24$

$36=3x$

$x=12$

$\log_7(x-3)+\log_7(x+1)=\log_7(5)$

$\log_7(x-3)(x+1)=\log_7(5)$

$(x-3)(x+1)=5$

$x^2-2x-3=5$

$x^2-2x-8=0$

$(x-4)(x+2)=0$

$x=4, \cancel{x=-2}$

$\log_5(x+12)+\log_5(x-12)=2$

$\log_5(x+12)(x-12)=2$

$(x+12)(x-12)=5^2=25$

$x^2-144=25$

$x^2=169$

$x=\pm 13$ only $x=13$ is a valid solution

8) $3\log A + 5\log B - 2\log C$
$= \log A^3 + \log B^5 - \log C^2$
$= \log \dfrac{A^3B^5}{C^2}$

$5\log x + 4\log y - \frac{1}{3}\log z$
$= \log x^5 + \log y^4 - \log \sqrt[3]{z}$
$= \log \dfrac{x^5y^4}{\sqrt[3]{z}}$

$\log x - 4\log y + 3\log z - 2\log w$
$= \log x - \log y^4 + \log z^3 - \log w^2$
$= \log \dfrac{xz^3}{y^4w^2}$

$\frac{1}{2}\ln A + \frac{1}{3}\ln B - \frac{1}{4}\ln C$
$= \ln \sqrt{A} + \ln \sqrt[3]{B} - \ln \sqrt[4]{C}$
$= \ln \dfrac{\sqrt{A}\,\sqrt[3]{B}}{\sqrt[4]{C}}$

---

9) $\ln\left(\dfrac{x^2y^3}{z^5}\right) = \ln x^2 + \ln y^3 - \ln z^5$
$= 2\ln x + 3\ln y - 5\ln z$

$\log_8\left(\dfrac{3x^5}{\sqrt{y}}\right) = \log_8 3 + \log_8 x^5 - \log_8 \sqrt{y}$
$= \log 3' + 5\log_8 x - \frac{1}{2}\log_8 y$

$$\log\left(\frac{x\sqrt[4]{y^3}}{z^2}\right) = \log x + \log y^{\frac{3}{4}} - \log z^2$$
$$= \log x + \tfrac{3}{4}\log y - 2\log z$$

$$\log_9 \frac{xy^7}{z^2 w} = \log_9 x + \log_9 y^7 - \log_9 z^2 - \log_9 w$$
$$= \log_9 x + 7\log_9 y - 2\log_9 z - \log_9 w$$

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10) $f(x) = \log_6(2x+1) - 3$

$$y = \log_6(2x+1) - 3$$
$$x = \log_6(2y+1) - 3$$
$$x+3 = \log_6(2y+1)$$
$$6^{x+3} = 2y+1$$
$$6^{x+3} - 1 = 2y$$
$$\frac{6^{x+3}-1}{2} = y = f^{-1}(x)$$

$$f(x) = 3\ln(7x) - 4$$
$$y = 3\ln(7x) - 4$$
$$x = 3\ln(7y) - 4$$
$$x+4 = 3\ln(7y)$$
$$\frac{x+4}{3} = \ln(7y)$$
$$e^{\frac{x+4}{3}} = 7y$$
$$\tfrac{1}{7}e^{\frac{x+4}{3}} = y = f^{-1}(x)$$

$f(x) = 8^{4x-5}+1$ 10

$y = 8^{4x-5}+1$

$x = 8^{4y-5}+1$

$x-1 = 8^{4y-5}$

$\log_8(x-1) = 4y-5$

$\log_8(x-1)+5 = 4y$

$\frac{\log_8(x-1)+5}{4} = y = f^{-1}(x)$

$f(x) = 6e^{1-5x}$

$y = 6e^{1-5x}$

$x = 6e^{1-5y}$

$\frac{x}{6} = e^{1-5y}$

$\ln\left(\frac{x}{6}\right) = 1-5y$

$5y = 1-\ln\left(\frac{x}{6}\right)$

$y = \frac{1-\ln\frac{x}{6}}{5} = f^{-1}(x)$

---

11) $(f\circ g)(x) = 2(\log_4 16)^3 - 1$

$(f\circ g)(x) = \log_6(x^4-5-2) = \log_6(x^4-7)$

$(f\circ g)(x) = 4\log_5(5^{x-3})-2 = 4(x-3)-2 = 4x-14$

$(f\circ g)(x) = \log_2\left(3\left(\frac{2x-5}{3}\right)+5\right) = \log_2((2x-5)+5)$

$= \log_2 2x$

$y = ab^x$

solve for a: $y = 200b^x$

solve for b: $180 = 200b^3$

$\frac{9}{10} = b^3$

$\left(\frac{9}{10}\right)^{\frac{1}{3}} = b$

$y = 200\left(\left(\frac{9}{10}\right)^{\frac{1}{3}}\right)^x = 200\left(\frac{9}{10}\right)^{\frac{x}{3}}$

$y = 200\left(\frac{9}{10}\right)^{\frac{7}{3}}$

$y = a \cdot b^x$

solve for a: $y = (.05)b^x$

solve for b: $.08 = .05b^{10}$

$\frac{8}{5} = b^{10}$

$\left(\frac{8}{5}\right)^{\frac{1}{10}} = b$

$y = (.05)\left(\left(\frac{8}{5}\right)^{\frac{1}{10}}\right)^x$

$y = (.05)\left(\frac{8}{5}\right)^{\frac{x}{10}}$

$y = .05\left(\frac{8}{5}\right)^{\frac{24}{10}}$

$= .1545$ or 15.45%

12 $F = 1000\left(e^{(.08)(5)}\right) \approx \$1491.82$ 11

$y = ab^x$

solve for a: $y = 200b^x$

solve for b: $160 = 200b^5$

$\frac{4}{5} = b^5$

$\left(\frac{4}{5}\right)^{\frac{1}{5}} = b$

$y = 200\left(\left(\frac{4}{5}\right)^{\frac{1}{5}}\right)^x = 200\left(\frac{4}{5}\right)^{\frac{x}{5}}$

$100 = 200\left(\frac{4}{5}\right)^{\frac{x}{5}}$

$\frac{1}{2} = \left(\frac{4}{5}\right)^{\frac{x}{5}}$

$\log\frac{1}{2} = \log\left(\frac{4}{5}\right)^{\frac{x}{5}}$

$\log\frac{1}{2} = \frac{x}{5}\log\frac{4}{5}$

$\frac{5\log\left(\frac{1}{2}\right)}{\log\left(\frac{4}{5}\right)} = x$