MAT 118 Spring 2017   Name   _____
Midterm #1   ID#   _____

Please show your work.
And remember: no calculators!
The test is out of 100 points.

1. [17 pts] Consider the following preference schedule for an election between candidates A, B, C:

# voters:	6	3	1	2
1st	A	C	B	C
2nd	B	B	C	A
3rd	C	A	A	B

   (a) [2 pts] Which candidate wins using the plurality method?

   (b) [5 pts] Use the weighted Borda count method to determine the outcome, in which 1st place votes are 5 points, 2nd place votes are 4 points, and 3rd place votes are 1 point.

(c) [5 pts] Use the plurality-with-elimination method to determine the outcome.

(d) [5 pts] Finally, apply the method of pairwise comparisons to determine the outcome.

2. [5 pts] Is there a Condorcet candidate in the following election? If so, who is it? Even if your answer is "no," be sure to show the work that substantiates your claim.

# voters:	6	5	4
1st	A	C	B
2nd	B	D	A
3rd	C	A	C
4th	D	B	D

3. [13 pts] Consider the weighted voting system $[17; 7, 6, 4, 3]$ with players P_1, P_2, P_3, P_4.

(a) [5 pts] Does any player have veto power? If so, how many and which ones?

(b) [5 pts] List the winning coalitions and underline the critical players. Using this information compute the Banzhaf power distribution.
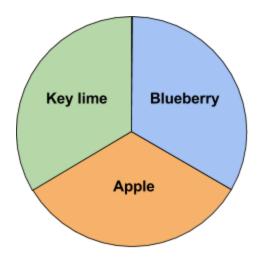
(c) [3 pts] How many sequential coalitions are there in this weighted voting system?

4. [10 pts] Consider the weighted voting system [500; 499, 498, 1].

   (a) [5 pts] List the sequential coalitions and underline the pivotal players. Then compute the Shapley-Shubik power distribution.

   (b) [5 pts] List the winning coalitions and underline the critical players. Using this information compute the Banzhaf power distribution.

5. [6 pts] In a weighted voting system with 3 players, the winning coalitions are $\{ P_1, P_2 \}$, $\{ P_1, P_3 \}$ and $\{ P_1, P_2, P_3 \}$. Compute the Shapley-Shubik power distribution.

6. [13 pts] Albert and Brenda are dividing a massive multi-flavored pie using the divider-chooser method. After flipping a coin, it is decided that Albert will be the divider. The pie has three flavors: apple, blueberry and key lime. Here's an illustration of the pie:

Albert likes apple and key lime equally, but likes blueberry twice as much as either of those other flavors. Brenda likes all three flavors equally.

(a) [3 pts] Make a table listing the values, relative to the total value of the cake, of each flavor according to each of Albert and Brenda. Use either fractions or percentages.

(b) [5 pts] Describe, with pictures and in words (i.e. fractions of flavors), **two** distinct ways that Albert can divide the pie. Remember that the division must be into two parts that Albert considers of equal value.

(c) [5 pts]Describe, for each of the two divisions you chose in part (b), the pieces that Brenda will choose, and how much they are worth to her.

7. [13 pts] In problem #6, suppose that we have an additional third player, Chandler. Chandler only likes blueberry. Now suppose the 3 players decide to use the Lone-divider method.

(a) [3 pts] Write a table, enlarging the table from 6(a), listing the values, relative to the total value of the pie, of each flavor according to the 3 players.

(b) [5 pts] Suppose Brenda is the divider, and that she divides the cake along the flavor lines, so that one piece is all apple, one is all blueberry, and one is all key lime. What are the bidding lists (i.e. fair share lists) of Albert and Chandler?

(c) [5 pts] Describe how this instance of the Lone-divider method might be completed.

8. [10 pts] Describe how the Lone-Chooser method may go with the pie in #6, in which Chandler is the chooser, and Albert is the divider when Albert and Brenda use the divider-chooser method at the start. How much value does each player end up with?

9. [13 pts] Alice and Bert are dividing a $10 sandwich which is half chicken parm and half vegetarian. Bert likes the chicken parm part four times as much as vegetarian part, while Alice is a strict vegetarian.

| Chicken Parm | Vegetarian |

(a) [3 pts] Make a table that records how much money the different flavor halves of the sandwich are worth to Alice and Bert, in dollars.

(b) [5 pts] Draw and describe a cut that Bert can make if he is acting as the divider in the divider-chooser method.

(c) [5 pts] After the cut, how much money is the piece that Alice chooses worth to her?