Quiz 4 MAT 118, Spring 2003

PRINT your Name: Solution

1. If you invest \$1000 at 8% annually, compounded monthly, how many months will it be until you double your money?

$$\log(1000)\left(1+\frac{.08}{12}\right) \qquad \frac{\log(2000)}{\log\left(1+\frac{.08}{12}\right)} \qquad \frac{\log(1000)}{\log\left(1+\frac{.08}{12}\right)}$$

$$\boxed{\frac{\log(2)}{\log\left(1+\frac{.08}{12}\right)}} \qquad \sqrt{1000+\frac{.08}{12}} \qquad \frac{1}{12}\log\left(1+\frac{.08}{12}\right)$$

Solution: Since we want to double our money, the future value should be \$2000. As above, the periodic rate is $\frac{.08}{12}$, the principle is \$1000, and the time is in months. Thus, we need to solve

$$2000 = 1000\left(1+\frac{.08}{12}\right)^t$$

for t. First, divide both sides by 1000 to get

$$2 = \left(1+\frac{.08}{12}\right)^t$$

and then take the logarithm of both sides. Using the fact that $\log(b^x) = x\log b$, we get

$$\log 2 = t\log\left(1+\frac{.08}{12}\right)$$

Now divide to get

$$t = \frac{\log 2}{\log\left(1+\frac{.08}{12}\right)}$$

This is 104.31 months, that is, just over 8 years and 8 months.

(Note that this question is exactly the same as on quiz 3, problem 3, except this time the correct answer was one of the choices.)

2. Suppose that at the end of each month, you put \$100 into an account that pays 8% annual interest, compounded monthly. How much money will be in the account at the end of 5 years?

$$\boxed{100\left(\frac{\left(1+\frac{.08}{12}\right)^{60}-1}{\frac{.08}{12}}\right)} \qquad \frac{\left(100+\frac{.08}{12}\right)^{60}+1}{\frac{.08}{12}} \qquad 100\left(\left(1+\frac{.08}{12}\right)^{60}\right)$$

$$1200\left(1+\frac{.08}{12}\right)^5 \qquad 100\left(\frac{\left(1+\frac{.08}{12}\right)^{60}}{1-\frac{.08}{12}}\right) \qquad \frac{100}{12}\log\left(1+\frac{.08}{12}\right)^{60}$$

Solution: This is exactly the formula for systematic savings, with the proper numbers plugged in. More specifically, we make a deposit of \$100 at the end of each month, so there should be a 100 in front. Since the annual rate is 8% and we are compounding monthly, the periodic rate is .08/12, and the time should be in months. 5 years is 60 months, so we have

$$100\left(\frac{\left(1+\frac{.08}{12}\right)^{60}-1}{\frac{.08}{12}}\right)\approx 7,347.68$$

The amount we put into the account is $1,200 a year for 5 years, or $6,000. We make nearly $1,350 in interest.

3. Suppose that at the end of each month, you put $100 into an account that pays 8% annual interest, compounded monthly. How many months will it take to have at least $2000 in the account?

$\dfrac{\log 301}{\log 1+\frac{.08}{12}}$	$\boxed{\dfrac{\log\left(\frac{16}{120}+1\right)}{\log\left(1+\frac{.08}{12}\right)}}$	$\dfrac{\log(2000+\frac{.08}{12})}{1-\frac{.08}{12}}$
$\log\left(\dfrac{1+\frac{.08}{12}}{\log(100+\frac{.08}{12})}\right)$	$100\left(\dfrac{\left(1+\frac{.08}{12}\right)^{60}}{1+\frac{.08}{12}}\right)$	$\dfrac{1}{12}\log\left(2000+\dfrac{.08}{12}\right)^5$

Solution: I think I must be cursed on the third problem of these quizzes. Yet again, there was a typo and the correct answer didn't appear. I've changed the answer above, and will give the correct solution here. Of course, everyone got full credit anyway.

Since we want to know the time to have $2000, we use the systematic savings formula to get

$$2000=100\left(\frac{\left(1+\frac{.08}{12}\right)^{t}-1}{\frac{.08}{12}}\right)$$

To solve this for t (which is the time in months), we divide both sides by 100 to obtain $20=\left(\frac{\left(1+\frac{.08}{12}\right)^{t}-1}{\frac{.08}{12}}\right)$, and then multiply both sides by .08/12: $\frac{1.6}{12}=\left(1+\frac{.08}{12}\right)^t-1$

Now add 1 to both sides, and take the logarithm (I also rewrote $\frac{1.6}{12}$ as $\frac{16}{120}$, more reduction is possible):

$$\log\left(1+\frac{16}{120}\right)=t\log\left(1+\frac{.08}{12}\right)$$

Solving for t gives

$$t=\frac{\log\left(1+\frac{16}{120}\right)}{\left(1+\frac{.08}{12}\right)}\approx 18.837$$

So, it will take 19 months to have more than $2000.