Quiz 3 MAT 118, Spring 2003

PRINT your Name: Solution

1. You will need \$2400 in cash two years from now. Your parents tell you that if you give them some amount of money now, they will pay you 10% annual simple interest on it, with no compounding. How much money do you need to give them in order to have the \$2400 in two years?

   Solution: First, we recall that for simple interest, $F = P(1 + r \cdot t)$. In our case, we know the future value F is \$2400, that the annual rate is 10%, and the time is 2 years. We want to know the principle P. Since both the rate and the time are given in years, all our units match and there is no need for conversion. Thus, we need to solve

   $$2400 = P(1 + (.10)(2)) = 1.2P$$

   for P, giving

   $$P = \frac{2400}{1.2} = 2000$$

   So we need to give them \$2000 now to have \$2400 in two years.

2. If you invest \$1000 in a bank account that pays 8% annual interest, compounded monthly, how much will there be in the account after 3 years?

   $\$1000\left(1 + \frac{8}{12}\right)^3$ $\quad$ $\$1000\,(1 + .08)^{36}$ $\quad$ $\boxed{\$1000\left(1 + \frac{.08}{12}\right)^{36}}$

   $\$\left(1000 + \frac{.08}{12}\right)^3$ $\quad$ $\$1000 + \left(\frac{.08}{12}\right)^{36}$ $\quad$ $\$1000\left(\frac{8}{12}\right)^3$

   Solution: Our principle is \$1000. Since the account is compounded monthly, our periodic interest rate is $\frac{.08}{12}$ (there are 12 months in a year). We also need to express our time in months, and 3 years is 36 months. Thus, the amount is expressed as

   $$1000\left(1 + \frac{.08}{12}\right)^{36}$$

3. If you invest \$1000 at 8% annually, compounded monthly, how many months will it be until you double your money?

   $\log(1000)\left(1 + \frac{.08}{12}\right)$ $\quad$ $\frac{\log(2000)}{\log\left(1 + \frac{.08}{12}\right)}$ $\quad$ $\boxed{\frac{\log(2)}{\log\left(1 + \frac{.08}{12}\right)}}$

   $\frac{\log(1000)}{\log\left(1 + \frac{.08}{12}\right)}$ $\quad$ $\sqrt{1000 + \frac{.08}{12}}$ $\quad$ $\frac{1}{12}\log\left(1 + \frac{.08}{12}\right)$

Solution: Since we want to double our money, the future value should be \$2000. As above, the periodic rate is $\frac{.08}{12}$, the principle is \$1000, and the time is in months. Thus, we need to solve

$$2000 = 1000\left(1 + \frac{.08}{12}\right)^t$$

for t. First, divide both sides by 1000 to get

$$2 = \left(1 + \frac{.08}{12}\right)^t$$

and then take the logarithm of both sides. Using the fact that $\log(b^x) = x \log b$, we get

$$\log 2 = t \log\left(1 + \frac{.08}{12}\right)$$

Now divide to get

$$t = \frac{\log 2}{\log\left(1 + \frac{.08}{12}\right)}$$

This is 104.31 months, that is, just over 8 years and 8 months.

(Note that on the original quiz, the correct answer had a typo, so everyone got full credit on this problem, no matter which choice they picked. Duh.)