You have 70 minutes to complete this exam. You may use a calculator. You may **NOT** use any books or notes. Please SHOW YOUR WORK and EXPLAIN YOUR REASONING wherever possible. This midterm has 9 questions, for a total of 180 points. Good luck!

1. (15 points) If one card is drawn from a deck of cards, what is the probability of drawing neither a king nor a spade?

2. (a) (10 points) In how many ways can a teacher decide which 2 of the 10 children in his first grade class will do "show and tell" today?

   (b) (10 points) A coin is flipped five times in a row. How many outcomes are possible? What is the probability that we get Heads five times in a row ?

3. (10 points) In how many ways can a first grade teacher pick from among his 10 students one child to play a DRAGON and one child to play an ELF in the school play?

4. (15 points) Supposing that there are equal chances of a boy or a girl being born, what is the probability that at least one of the first three babies born next Saturday morning at the St. Charles Hospital will be a girl?

5. (10 points) Suppose that 1% of the population of the United States is hypoglycemic. In a randomly selected group of 99 people from the United States, is the chance that at least one of the people will be hypoglycemic smaller, equal or larger than 99%?

6. (30 points) Determine whether the following graph has an Eulerian circuit, and if it does use Fleury's algorithm to find one and sketch it carefully on the picture.
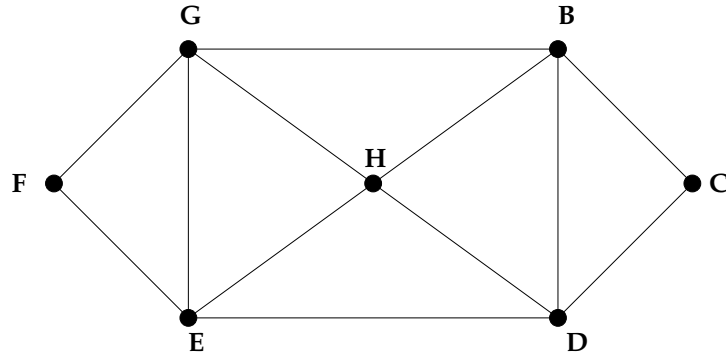
7. Answers the following questions and justify your answer:

    (a) (10 points) Does $K_{3,3}$ have any Eulerian paths?

    (b) (10 points) Does $K_{2,3}$ have any Eulerian circuits? How about any Eulerian paths ?

8. (30 points) Suppose that you plan a trip that includes stops in 6 cities: New York, Denver, Chicago, Kansas City, Dallas, and Miami. Assume that you live in one of these cities, so that your planned trip must begin and end in the same city. You would like your trip to be as short as possible.

   After checking distances between each of these cities you come up with the following weighted graph:
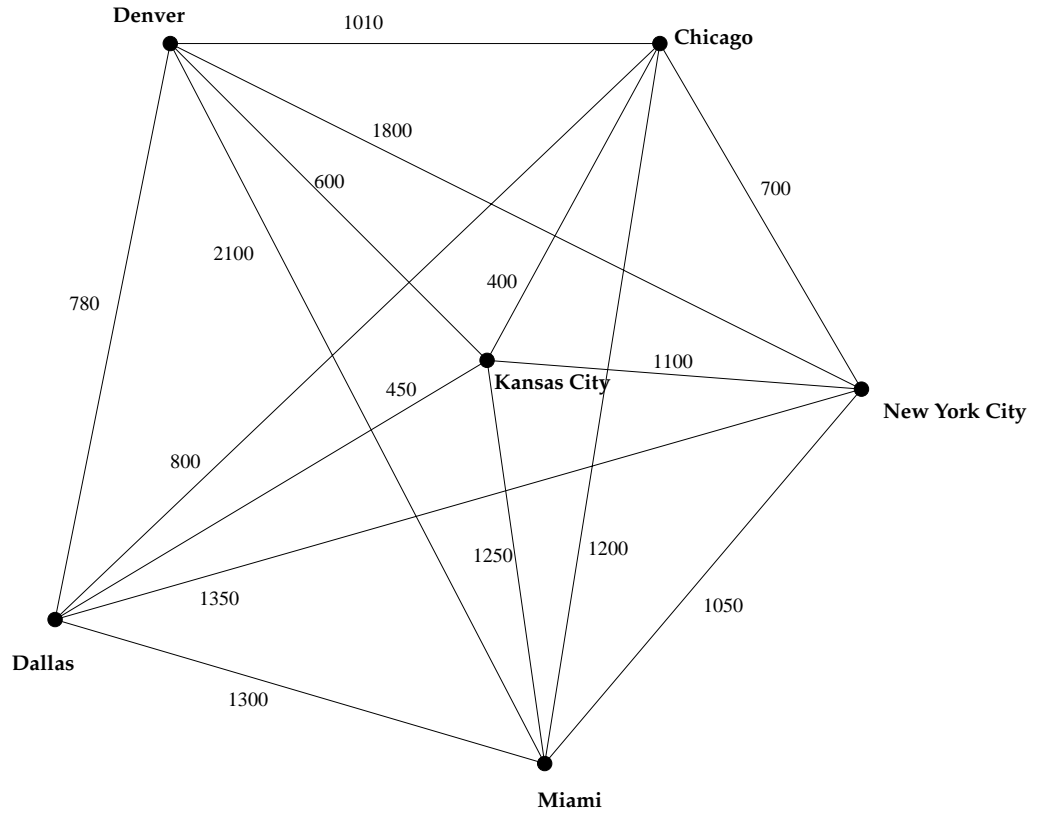
Starting with the Denver vertex, use the nearest neighbor algorithm to find a (potentially short) Hamiltonian circuit. What is the total distance of this circuit? Carefully sketch the circuit on the above picture.

Continue here the solution of the previous problem.

9. Decide whether each of the following statements is true or false:

(a) (5 points) If when flipping a coin three times in a row we denote

- A = the event that exactly two of the flips are Heads
- B = the event that the first flip is Tails,

then A and B are independent events.

(b) (5 points) When we roll two dice, a white one and a red one, then

$$P(\text{the white is six OR the red is six}) = P(\text{the white is a 6}) + P(\text{the red is a six}).$$

(c) (5 points) The graph $K_{3,3}$ has 6 vertices and 9 edges.

(d) (5 points) The nearest neighbor algorithm always produces the shortest Hamiltonian circuit.

(e) (5 points) There exist planar maps that cannot be colored with four colors, but they have hundreds of territories.

(f) (5 points) If every vertex of a connected graph has even degree, then that graph has an Eulerian circuit.