Solutions to First Exam

MAT 118, Spring 2003

1.*(75 points)* The seven members of a spelunking club want to choose one member to go first in exploring a dangerous cave. There are four volunteers: Abelard, Bobo, Cecilia, and Duncan. The votes of the club members are summarized in the table at right.

volunteer	preferences						
Abelard	1	2	4	3	2	3	3
Bobo	3	1	3	4	3	2	2
Cecelia	2	3	1	1	4	4	4
Duncan	4	4	2	2	1	1	1

a. Which volunteer would win in a plurality vote?

Solution: The tally of first-place preferences, which is all we look at in a plurality vote, is Abelard 1, Cecelia 2, Bobo 1, and Duncan 3. Thus **Duncan** is the winner.

b. Which volunteer would win in a plurality vote with a runoff between the top two finishers? (show your work)

Solution: The top two finishers are Cecelia and Duncan. The first four voters prefer Cecelia to Duncan, so **Cecelia** wins, 4 to 3.

c. Which volunteer would win the Borda count? (show your work)

Solution: Recall that we give 4 points for a first-place vote, 3 for a second, 2 points for a third, and 1 point for a fourth.

Abelard: $1 \times 4 + 2 \times 3 + 3 \times 2 + 1 \times 1 = 17$

Bobo: $1 \times 4 + 2 \times 3 + 3 \times 2 + 1 \times 1 = 17$

Cecelia: $2 \times 4 + 1 \times 3 + 1 \times 2 + 3 \times 1 = 16$

Duncan: $3 \times 4 + 2 \times 3 + 0 \times 2 + 2 \times 1 = 20$

So **Duncan** is the winner of the Borda count.

d. Which volunteer, if any, is the Condorcet winner? (show your work).

Solution: We know (from part b) that Cecelia beats Duncan in a head-to-head race, so Duncan cannot be the Condorcet winner.

But Abelard beats Cecelia (4 to 3), so she is not the Condorcet winner, either.

Duncan wins against Abelard (5 to 2) and against Bobo (also 5 to 2), so neither of them can be the Condorcet winner.

Thus, there can be **no Condorcet winner**.

e. Each club member is asked who s/he approves of for the job, and the responses are: Abelard & Cecelia; Abelard & Bobo; Cecelia; Cecelia & Duncan; Duncan; Duncan; Bobo & Duncan. Who wins the approval vote?

Solution: Duncan gets 4 approval votes, Cecelia gets 3, Bobo 2, and Abelard 2. Thus **Duncan** wins the approval vote.

2. *(20 points)* You put \$1000 in a bank account that pays 6% annual interest, compounded monthly. How much will be in the account in three years?

Solution: This is a straightforward calculation. We are asked for the future value of \$1000 invested for 3 years. Using the formula $F = P\left(1 + \frac{r}{n}\right)^T$, remembering that T is in the same units as the compounding (so $T = 12 \times 3 = 36$), we have

$$F = 1000\left(1 + \frac{.06}{12}\right)^{36} = \$1,196.68$$

3. *(20 points)* You put \$1000 in a bank account that pays 6% annual interest, compounded monthly. How long will it take for the account to reach at least \$2000?

Solution: Here we are given both the present value (\$1000) and the future value (\$2000), and are asked to solve for the time. So, we have

$$2000 = 1000\left(1 + \frac{.06}{12}\right)^T$$

Divide both sides by 1000 to get

$$2 = \left(1 + \frac{.06}{12}\right)^T$$

Now take the logarithm of both sides, and use a property of $\log$ to get

$$\log 2 = T \log\left(1 + \frac{.06}{12}\right)$$

Finally, solve for T:

$$T = \frac{\log 2}{\log\left(1 + \frac{.06}{12}\right)} \approx 138.975$$

So, after 139 months (11.58 years), there will be more than \$2000 in the account.

4.*(20 points)* Which earns more money (circle your answer, and justify it below):

a. An account earning 10% annual interest, compounded yearly.

[b]. An account earning 9.75% annual interest, compounded monthly.

c. An account earning 9.5% annual interest, compounded daily.

d. Putting your money in a jar labelled "magic money multiplier".

Solution: You might be tempted to answer **d.**, but the jar is only labelled as a magic money multipler, the label could be wrong. Or, perhaps it is right, but it multiplies the amount of money you have by zero!

So, we should check the other possibilities. We can either calculate the APY on each, or if you like, just calculate how much you would earn in a year if you invest the same amount in each.

a. This compounds yearly, so its annualized yield is 10%.

b. Here the APY is given by $\left(1+\frac{.0975}{12}\right)^{12}-1 \approx .101977220$, that is, almost 10.2%.

c. The APY is $\left(1+\frac{.095}{365}\right)^{365}-1 \approx .099645274$, or 9.96%

Thus, investing at 9.75% compounded monthly is the best deal.

5.*(20 points)* Which yields the most money (circle your answer, and justify it below):

a. Monthly deposits of \$100 at 12% annual interest, compounded monthly for 12 years.

b. Monthly deposits of \$200 at 6% annual interest, compounded monthly for 12 years.

[c]. Monthly deposits of \$100 at 6% annual interest, compounded monthly for 24 years.

Solution: We can get a feel by just thinking a bit: **a** puts in a total of $\$100 \times 12 \times 12 = \$14,400$, **b** puts in a total of $\$200 \times 12 \times 12 = \$28,800$, and **c** puts in a total of $\$100 \times 12 \times 24 = \$28,800$. We would need a huge interest rate for **a** to come out ahead, so it is down to **b** and **c**. But **c** has twice as long for the interest to accumulate, so I expect **c** to be the right answer. We can check this by explicitly calculating the accumulations.

$$\$100\left(\frac{\left(1+\frac{.12}{12}\right)^{144}-1}{\frac{.12}{12}}\right) \approx \$31,906.15$$

$$\$200\left(\frac{\left(1+\frac{.06}{12}\right)^{144}-1}{\frac{.06}{12}}\right) \approx \$42,030.03$$

$$\$100\left(\frac{\left(1+\frac{.06}{12}\right)^{288}-1}{\frac{.06}{12}}\right) \approx \$64,111.58$$

So, **c** is certainly the best choice.

6. *(20 points)* Mma Makutsi takes out an amortized loan of \$30,000 at 6% annual interest, compounded monthly. Assuming she pays it back by making equal payments at the end of each month for 24 years, how much is Mma Makutsi's monthly payment?

Solution: Note that we are given the loan amount (P) and want to calculate the regular payment (R), so using the formula, we have

$$30000 = R\left(\frac{1-\left(1+\frac{.06}{12}\right)^{-288}}{\frac{.06}{12}}\right)$$

and want to solve for R. Multiplying out the right hand side, we get

$$30000 \approx 152.4441214R$$

so $R \approx \$196.79$ per month.