Last time:

We discussed the notion of wallpaper symmetry, and that there are only four main types of rotational symmetries in the wallpaper groups.
Here are the rotational symmetries that work:

- $180°$ rotation
- $120°$ rotation
- $90°$ rotation
- $60°$ rotation

These rotational symmetries come from pieces of wallpaper shaped like triangles, squares, hexagons, and octagons.
Why are these the only four shapes that work?
Since we are dealing with wallpaper, the patterns of the wallpaper must have translational symmetry.
Here's an examples of translational symmetry.

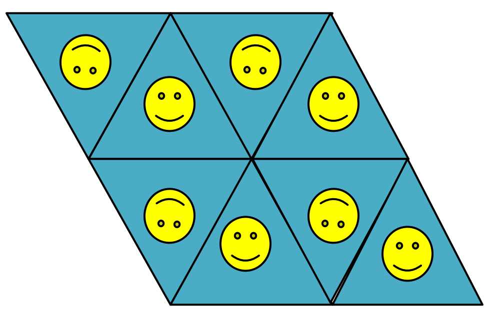
```
/* /* /* /* /* /
 /* /* /* /* /* /
  /* /* /* /* /* /
   /* /* /* /* /* /
    /* /* /* /* /* /
```

What are regular polygons?

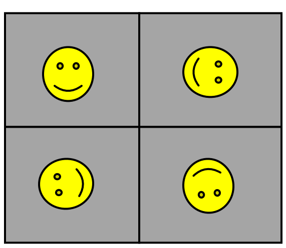
A regular polygon is a polygon, which is equiangular equilateral.

Now let's consider some shapes, and try to duplicate the shape each time in order to tile the plain.

1. *Equilateral triangles*

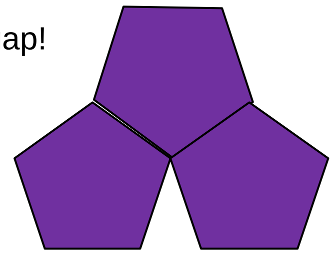


2. *Squares*

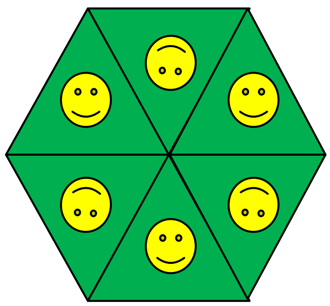


3. <u>Pentagon</u>

Will not work, there is a gap!



4. <u>Hexagons</u>



Notice that hexagon is constructed from equilateral triangles; therefore, it is the same as before.

5. <u>Octagons</u>

Will not work for the same reason pentagons do not work!

We can keep doing this notice, which ones work.

<u>Platonic Solids</u>

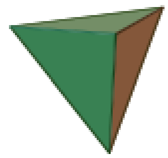
Let's take 12 pentagon, try to draw them so that most of the plain space is taken up. Then if we were to take the gaps and close them, we would get a "platonic solid".

What is a platonic solid?

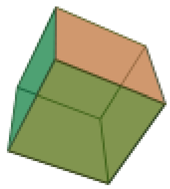
A Platonic solid is a convex polyhedron that is regular. The faces of a Platonic solid are all congruent regular polygons. Platonic solids have a unique property that the faces, angles, and edges of each solid are all congruent.

There are exactly five Platonic solids, as follows:

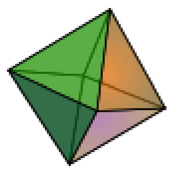
- Tetrahedron Cube



- Cube or Hexahedron

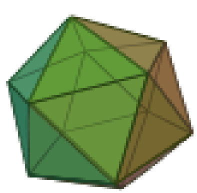


- Octahedron



- Dodecahedron

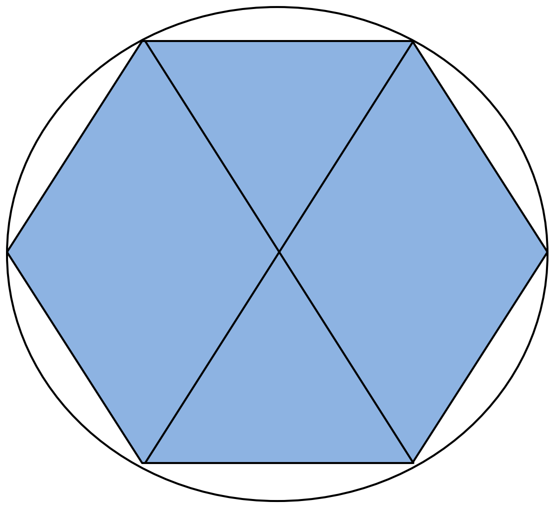
- Icosahedron



How do we find the measure of an angle of any polygon?

Example: What's the measure of an angel in a hexagon?

Notice that we can inscribe the hexagon in to a circle and draw diagonal lines.



The diagonals divide the circle into six equal pieces, so we have $\frac{360°}{6} = 60°$. That is the measure of the angle in the triangle, where all the vertices meet. Now we look at the triangle and subtract $180° - 60° = 120°$ to get the angle of the hexagon.

In general, here is the formula:

$$180° - \frac{360°}{n} = 2x$$

Where n is the number of sides in a polygon, and $2x$ represents the angle we are looking for.