MAE 301

2/12

In the previous class we touched upon trinomial factoring, and here's an example of something students might do, which is bad:

$$x^2 - 3x - 4 = 0$$
$$x^2 - 3x = 4$$
$$x(x - 3) = 4$$

At this point they figure since: $2 * 2 = 4$
they can say: $x = 2$ $x - 3 = 2$
$x = 5$

If students had no trouble with logic and reasoning, we could simply give them the axioms and send them on their merry way. Needless to say, this is not the case.

Our goal is to make students proficient, and ideally to have a deep conceptual understanding.

Goals:

- Procedural Fluency - being good at solving problems. (*more often than not, too much focus is placed here in the classroom).
- Strategic competency – knowing how to approach problems.
- Adaptability – ability to modify old procedure & strategy to new problems.
- Mathematical disposition – knowing when to use math to solve a problem.

Again, there is often too much focus on the procedure, basically "here's a problem; this is how you do it."

Of course the teacher is not entirely to blame. Testing is a heavy factor for various reasons. For one, the test content itself focuses on procedure, testing students specifically on their procedural fluency. Also, many teachers feel that the most certain way to get students to achieve good scores on the test is to simply drill the procedure rather than teaching the concepts, and letting the procedure follow naturally.

We as teachers will need to emphasize more besides the process.

Most students are easily capable of solving this problem:

$$3x + 5 = 8$$

However if they are given this problem instead:

$$ax + b = 0\,, \quad (a \neq 0)$$

Or:

$$\pi x + \sqrt{2} = 7$$

They will have difficulties because, even though we know all 3 of those to be the same, to them, things like a, b, and $\sqrt{2}$ are not numbers, so it does not fall within the same procedure they're familiar with.

Aside: formulae are pedagogically unsound!

To solve: $ax^2 + bx + c = 0$

We know: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

But how would you concoct a lesson for:

<u>Completing the square</u> (the quadratic formula)

Starting with an example:

$x^2 + 4x + 4 = 0$ Is an easy example.

Here $a = 1$ is good, and 4 is good because $\frac{4}{2}$ is our friend.

We already know by distribution:

$$\begin{aligned} &(x + a)(x + b) \\ &= x(x + b) + a(x + b) \\ &= x^2 + bx + ax + ab \end{aligned}$$

$$= x^2 + (a+b)x + ab$$

So it can be seen from here that to factor $x^2 + Mx + E$ you need to find a, b so that

$$ab = E$$
$$a + b = M$$

Students will say an example like "doesn't factor"; when what they are really saying is that there are no $a, b \in \mathbb{Z}$ so that $ab = 1$ and $a + b = 4$.

In regards to $x^2 + 4x + 1$:

How can we know $\exists x \in \mathbb{R}$ solving it?

- Graph it?
- Experiment with values for x. (this is technically the intermediate value theorem) (also, basically the same as graphing it).
- …
- ...

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