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MAE301 notes for **1/27/09**

What is Mathematics?

- Numbers?
- Abstract reasoning, problem solving, logic, proofs?
- Finding patterns?

The answer is that there is all of the above, and more. Many things you find in math you will also find in almost all other fields of study, such as the various sciences, economics, and architecture. As a group, we cannot give one set definition for mathematics, but we do know when we see it.

In grade school, we learn numbers, fractions, calculations, equations, geometry, statistics, probability, sets, etc…

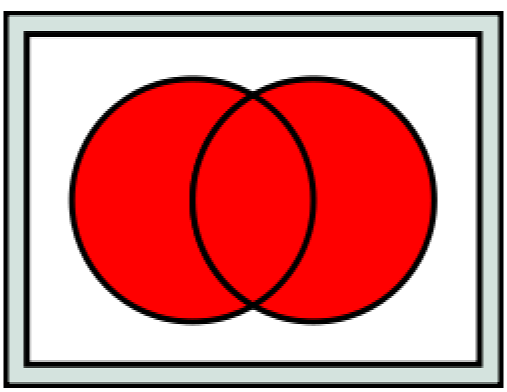
What is a set?

A set is a collection of objects. More specifically, we have two different types of sets;

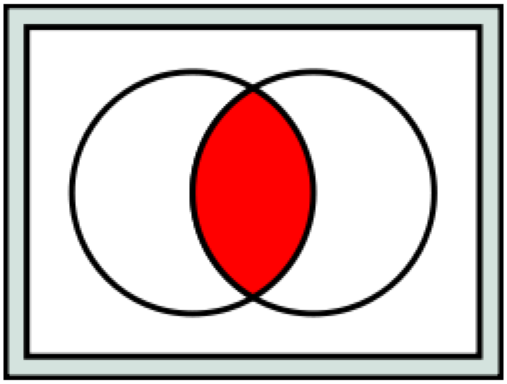
1. Enumeration (listing): $\{1, 2, shoe, cat\}$
2. A rule: $\{x \mid x^2 > 7\}$

Both 1 and 2 are examples of how a set can be described.

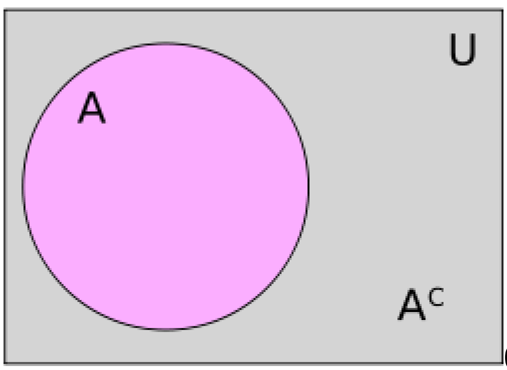
$A \cup B$ means (where S is any set) $\{x\epsilon\ S \mid x\epsilon\ A\ or\ x\epsilon\ B\}$ For example, if set A = {1,3,5,7} and set B = {2,4,6,8}, $A \cup B = \{1,2,3,4,5,6,7,8\}$


This is a Venn diagram representing the union of two sets A (on left) and B (on right). (http://en.wikipedia.org/wiki/Union_(set_theory))

$A \cap B$ means (where S is any set) $\{x\epsilon\ S \mid x\epsilon\ A\ and\ x\epsilon\ B\}$ For example, if set A = {1,3,5,7} and set B = {2,4,6,8}, $A \cap B = \{\emptyset\}$. If A = {1,3,5,7} and B = {1,2,3,4}, then $A \cap B = \{1,3\}$.


This is a Venn diagram representing the intersection of two sets A (on left) and B (on right). (http://en.wikipedia.org/wiki/Intersection_(set_theory))

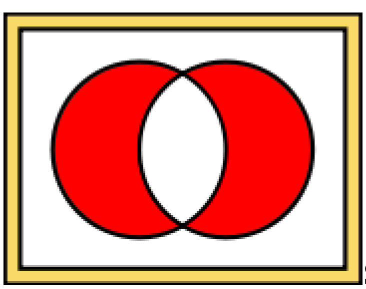
If we have a set A, and we want to find the compliment of A, this would be all other elements in a universe that are not in A. (A **universe** (U) is a class that contains all the entities one wishes to consider in a given situation. (Wikipedia)) Symbolically, the compliment of A is represented as: $A^c = U\backslash A$.



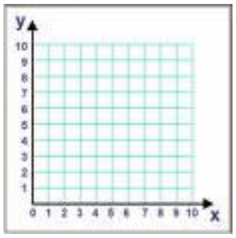
(http://en.wikipedia.org/wiki/Complement_(set_theory))

In set theory, we also have the symmetric difference, which is represented by:

$A\Delta B = (A\backslash B) \cup (B\backslash A)$ This means everything in A but not in B union with everything in B but not in A.


Set A (on left) and set B (on right). (http://en.wikipedia.org/wiki/Symmetric_difference)

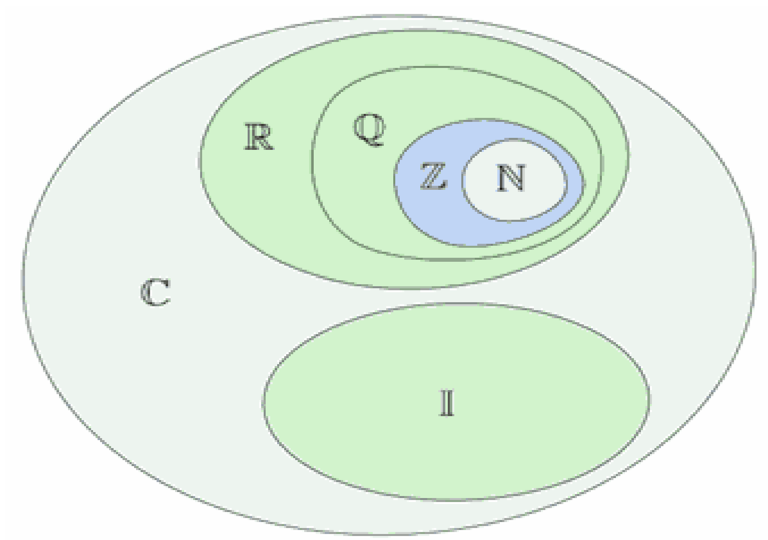
$A \times B = \{(a, b) \mid a\epsilon\ A\ and\ b\epsilon\ B\}$. This represents a cartesian product, which is a direct product of sets. An example of a Cartesian product that we see a lot in math classes would be $\mathbb{R} \times \mathbb{R}$, which is a graph of the real numbers on an x and y axis.



You will also find many examples of the Cartesian product in multi-variable calculus classes. (http://en.wikipedia.org/wiki/Cartesian_product)

("∪" is the symbol for the union of sets, "∩" is the symbol for the intersection of sets.)

Specific sets of numbers that we would encounter throughout our education would be: $\mathbb{N}$, which is the set of all Natural numbers; $\mathbb{Z}$, which is the set of all integers; $\mathbb{Q}$, which is the set of all rational numbers; $\mathbb{R}$, which is the set of all real numbers; $\mathbb{C}$, which is the set of all complex numbers; Irrational numbers (we'll label this as $\mathbb{Y}$); and Whole numbers (we'll label this as $\mathbb{W}$). We also know that $\mathbb{N} \subseteq \mathbb{W} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R} \subseteq \mathbb{C}\ and\ \mathbb{Y} \subseteq \mathbb{R} \subseteq \mathbb{C}$. $\mathbb{I}$ (which represents imaginary numbers) is only a subset of the complex numbers, as shown in the diagram below.



(http://www.mathsisfun.com/sets/number-types.html)

$$\mathbb{N} = \{1, 2, 3, \ldots\}$$

$$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

$$\mathbb{Q} = \{\frac{p}{q} \mid q \neq 0,\ \ p, q \in \mathbb{Z},\ \ (p, q) = 1\}$$

- Note: Rationals are solutions of linear equations $\{qx = p \mid p, q \in \mathbb{Z}\}$. Also, (p,q) = 1 means that p and q are relatively prime.

$\mathbb{R}$ is a little more difficult to define in terms of sets of numbers in part because of the fact that we define $\mathbb{Y}$ as $\mathbb{R} - \mathbb{Q}$, so to avoid ambiguity, we cannot define $\mathbb{R}$ as $\mathbb{Y} \cup \mathbb{Q}$. And so we define $\mathbb{R}$ as in terms of which we would describe it to students learning the real numbers in grade school:

$$\mathbb{R} = \{x \mid x\ has\ an\ infinite\ decimal\ expansion\}\ or\ \{points\ on\ a\ line\}$$

$$\mathbb{C} = \{\ x + iy \mid x, y \epsilon\ \mathbb{R}\ \}$$

$$\mathbb{Y} = \mathbb{R} - \mathbb{Q}$$

$$\mathbb{W} = \ \mathbb{N}\ \cup \{0\}$$

Other numbers that students may come across throughout their education would be imaginary numbers (when you square an imaginary number you will get a negative answer); $\{\ iy \mid y\ \epsilon \mathbb{R}, i = \sqrt{-1}\}$, algebraic numbers(which include all rational numbers and some irrational numbers); $\{\ x\epsilon\ \mathbb{R} \mid there'sa\ polynomial\ (with\ rational\ coefficients) P\ so\ that\ P(x) = 0\}$, and transcendental numbers, which are real numbers that are not algebraic.