Recall that if $I = [a, b] \subset \mathbb{R}$ then we defined $h_I : I \to \mathbb{C}$ by

$$h_I(x) = \begin{cases} |b-a|^{-\frac{1}{2}}, & \text{if } x \in [a, \frac{a+b}{2}) \\ -|b-a|^{-\frac{1}{2}}, & \text{if } x \in [\frac{a+b}{2}, b) \\ 0, & \text{if } x \notin [a, b) \end{cases}$$

The Collection of dyadic intervals $\mathcal{D}$ is defined by

$$\mathcal{D} := \{ [\frac{i}{2^j}, \frac{i+1}{2^j}) : i, j \in \mathbb{Z} \}$$

.

We have that $\{h_I : I \in \mathcal{D}\} \cup \{\chi_{[0,1]}\}$ is a basis for the vector space of Riemann integrable funcitons. Let

$$f(x) = \begin{cases} \epsilon^{-\frac{1}{2}}, & \text{if } x \in [\frac{1}{2}(1-\epsilon), \frac{1}{2}(1+\epsilon)] \\ 0, & \text{otherwise} \end{cases}$$

Assume $\epsilon = 2^{-n_0}$.

1. How many non-zero coefficients a_I would it take to write

$$f(x) = \sum_{\substack{I \in \mathcal{D} \\ I \subset [0,1]}} a_I h_I(x)$$

.

2. How many of them would be bigger than δ?
3. How many non-zero coefficients c_n would it take to write f in the Fourier basis?
4. How many of them would be bigger than δ?