MAT 319 and MAT 320

Practice Midterm I, Fall 2014.

This is a closed notes/ closed book/ electronics off exam.

You are allowed and encouraged to motivate your reasoning, but at the end your proofs should be formal logical derivations, whether proving that something holds for all, or proving that your example works.

You can use any theorem or statement proven in the book; please refer to it in an identifiable way, eg. “by the completeness axiom”, “by the definition of the limit”, etc.

Please write legibly and cross out anything that you do not want us to read.

Each problem is worth 25 points (but the problems are of variable difficulty!).

Name:					
Problem	1	2	3	4	Total
Grade					

Problem 1. Deduce from the Completeness Axiom that there exists a square root of a real number a if and only if $a \geq 0$.

Problem 2. Let $A \subset \mathbb{R}$ be a non-empty set, and let $c \in \mathbb{R}$.

a) Define what it means for c to be the least upper bound of A, i.e. that $c = \sup A$.

b) Prove that for all $\varepsilon > 0$ there exists $a \in A$ such that

$$\sup A - \varepsilon < a \leq \sup A.$$

c) Let (a_n) be a bounded increasing sequence. Show that (a_n) converges to $c = \sup\{a_n : n \in \mathbb{N}\}$.

Problem 3. Suppose that (a_n) and (b_n) are sequences of non-zero numbers. Define $c_n = a_n \cdot b_n$.

a) Suppose that (a_n) and (b_n) are both bounded. Prove that (c_n) is also bounded.
b) Suppose that (c_n) is bounded. Does this imply that both (a_n) and (b_n) are bounded (prove or give a counterexample)?

Problem 4. Prove rigorously that the sequence

$$a_n = \frac{n + 2^{-n}}{2n + \sqrt{n}}$$

converges, and compute its limit.