LAST NAME: FIRST NAME:

STONY BROOK ID NUMBER:

Problem	1	2	3	4	5	Total
Score						

MAT 319/MAT 320 Analysis Midterm 1

October 2, 2012

NO BOOKS OR NOTES MAY BE CONSULTED DURING THIS TEST.
NO CALCULATORS MAY BE USED.
Show all your work on these pages!
Total score = 100

1. (40 points) Here **N** represents the counting numbers $\{1, 2, 3, 4, \ldots\}$, **Z** represents the integers, **Q** the rational numbers and **R** the real numbers.

   a. Explain carefully why the equation $x + 5 = 1$ has no solution in **N**.

   b. Explain carefully why the equation $3x = 2$ has no solution in **Z**.

c. Explain carefully why the equation $x^2 = 7$ has no solution in $\mathbf{Q}$.

d. Explain carefully why the least upper bound property (the Completeness Axiom) guarantees that the equation $x^2 = 7$ has a solution in $\mathbf{R}$.

2. (15 points) Prove by induction that the sum of the first n odd integers is equal to n^2, i.e. that

$$1 + 3 + 5 + 7 + \cdots + (2n - 1) = n^2.$$

3. (15 points) For a pair (x, y) of real numbers, define $||(x, y)|| = |x| + |y|$. Prove carefully that

$$||(a + c, b + d)|| \leq ||(a, b)|| + ||(c, d)||.$$

4. (15 points) Here $\sin(x)$ is the usual sine function. Show that the sequence $a_1, a_2, a_3, \ldots$ defined by $a_n = \frac{\sin(n)}{n}$ converges, with limit 0.

5. (15 points) Suppose (s_n) is a sequence of positive numbers converging to the limit s. Prove that the sequence $(\sqrt{s_n})$ converges to $\sqrt{s}$. *Hint:* give separate proofs for $s = 0$ and $s > 0$.

END OF EXAMINATION