PRACTICE MIDTERM I

(1) Given a recursive formula

$$a_{n+1} = \sqrt{3 + 2a_n},$$
$$a_0 = 0,$$

find the limit $\lim_{n\to+\infty} a_n$

(2) Find the value of the continued fraction expression

$$1 + \cfrac{2}{1 + \cfrac{2}{1 + \cfrac{2}{1 + \cfrac{2}{\cdots}}}}$$

(3) Let

$$a_n = \frac{2^n}{n!},$$

compute the limit $\lim_{n\to+\infty} a_n$.

(4) Compute the sum

$$\sum_{n=0}^{\infty} \frac{2 + (-3)^n}{4^n}.$$

(5) Determine whether the following series converges or diverges

$$\sum_{n=0}^{\infty} \frac{\cos(\frac{3n\pi}{5})}{n^6 + 4}.$$

(6) Determine whether the following series converges or diverges

$$\sum_{n=0}^{\infty} \tan(\frac{n}{n^3 + 2n}).$$

(7) Determine whether the following series converges or diverges

$$\sum_{n=0}^{\infty} \frac{\ln(n)}{n + 3}.$$

(8) Determine whether the following series converges or diverges

$$\sum_{n=0}^{\infty} \frac{(-3)^n}{(2n)!}.$$

(9) Determine whether the following series converges or diverges

$$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{(\ln n)^2}.$$

(10) Determine whether the following series converges or diverges

$$\sum_{n=0}^{\infty} \frac{1}{n \cdot (\ln n)^4}.$$

(11) Determine whether the following series converges or diverges

$$\sum_{n=0}^{\infty} \frac{(-3)^n}{(2n)!}.$$

(12) Compute the radius of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n^2+1} \cdot (x-2)^n.$$

Then for what values of x, does this power series converge?

(13) For what values of x, does the power series

$$\sum_{n=0}^{\infty} \frac{1}{n \cdot 5^n} \cdot (2x+1)^n$$

converge?

(14) For what values of x, does the power series

$$\sum_{n=0}^{\infty} \frac{(n+1)!}{(2n+1)!} \cdot x^n$$

converge?