Spring 2017 MAT 536, Complex Analysis
Instructor: Samuel Grushevsky
Homework #9, due in class Wed April 12

Problem 1. Prove the Schwarz-Christoffel formula for the biholomorphism from a unit disk to the interior of a polygon with π-rational angles (*Ahflors gives a proof, but please try to a) do it on your own, b) if you read Ahlfors, explain it better*)

Problem 2. Let
$$D := \{z : |\operatorname{Im} z| < \pi/2\}$$
be the horizontal strip in $\mathbb{C}$. Construct a holomorphic function $f \in \mathcal{O}(D)$, which extends continuously to the boundary ∂D, and such that $f|_{\partial D}$ is bounded, but $f|_D$ is not bounded.

Problem 3. Suppose a holomorphic function $f \in \mathcal{O}(\Delta)$ on the unit disk extends continuously to the boundary $\partial\Delta$. Suppose moreover that $f(\partial\Delta) \subset \partial\Delta$. Prove that f is a rational function.

Problem 4. For any $0 \le r < R \le \infty$ define the corresponding annulus to be
$$A_{r,R} := \{z \in \mathbb{C} : r < |z| < R\}.$$
For which r, R, r', R' are the annuli $A_{r,R}$ and $A_{r',R'}$ biholomorphic?

Problem 5. Suppose $f : \mathbb{H} \to \mathbb{H}$ is a one-to-one holomorphic map of the upper half-plane to itself, and suppose some purely imaginary $a \in \mathbb{H}$ is fixed by f, i.e. $\exists a \in i\mathbb{R}_{>0}$ such that $f(a) = a$. Prove that $|f'(a)| \le 1$.