Spring 2017 MAT 536, Complex Analysis

Instructor: Samuel Grushevsky

Homework #12, due in class Wed May 3

Problem 1. Suppose f is a complex-valued harmonic function on Δ. Prove that if $|f|$ is constant in Δ, then f is constant in Δ.

Problem 2. Let $\Omega = \{z : 0 < \operatorname{Im} z < 1\}$. Which boundary points of Ω are Dirichlet regular?

Problem 3. Let $f \in \mathcal{C}^\infty(\partial\Delta)$ be a smooth function on the unit circle. How does the solution of the Dirichlet problem via Poisson formula compare to the upper envelope $\mathcal{U}_{\mathcal{F}_f}$ for the corresponding Perron family?

Problem 4. (double credit) Let Ω be the annulus around zero with inner radius 1 and outer radius 2. Let f be identically equal to a on the inner circle and equal to b on the outer circle. What is the upper envelope $\mathcal{U}_{\mathcal{F}_f}$ for the corresponding Perron family?