2-14. Let M be a topological manifold, and let $\mathcal{U}$ be a cover of M by precompact open sets. Show that $\mathcal{U}$ is locally finite if and only if each set in $\mathcal{U}$ intersects only finitely many other sets in $\mathcal{U}$. Give counterexamples to show that the conclusion is false if either precompactness or openness is omitted from the hypotheses.

2-16. Suppose M is a topological space with the property that for every open cover $\mathcal{X}$ of M, there exists a partition of unity subordinate to $\mathcal{X}$. Show that M is paracompact.

2-18. Let M be a smooth manifold, let $B \subset M$ be a closed subset, and let $\delta : M \to \mathbb{R}$ be a positive continuous function.
(a) Using a partition of unity, show that there is a smooth function $\tilde{\delta} : M \to \mathbb{R}$ such that $0 < \tilde{\delta}(x) < \delta(x)$ for all $x \in M$.
(b) Show that there is a continuous function $\psi : M \to \mathbb{R}$ that is smooth and positive on $M \setminus B$, zero on B, and satisfies $\psi(x) < \delta(x)$ everywhere. [Hint: Consider $1/(1+f)$, where $f : M \setminus B \to \mathbb{R}$ is a positive exhaustive function.]

3-1. Suppose M and N are smooth manifolds with M connected, and $F : M \to N$ is a smooth map such that $F_* : T_pM \to T_{F(p)}N$ is the zero map for each $p \in M$. Show that F is a constant map.

3-3. If a nonempty smooth n-manifold is diffeomorphic to an m-manifold, prove that $n = m$.

3-5. Consider $\mathbb{S}^3$ as a subset of $\mathbb{C}^2$ under the usual identification of $\mathbb{C}^2$ with $\mathbb{R}^4$. For each $z = (z^1, z^2) \in \mathbb{S}^3$, define a curve $\gamma_z : \mathbb{R} \to \mathbb{S}^3$ by $\gamma_z(t) = (e^{it}z^1, e^{it}z^2)$.
(a) Compute the coordinate representation of $\gamma_z(t)$ in stereographic coordinates, and use this to show that γ_z is a smooth curve
(b) Compute $\gamma_z'(t)$ in stereographic coordinates, and show that it is never zero.

3-8. Let M be a smooth manifolds and $p \in M$. Let $\mathcal{C}_p$ denote the set of smooth curves $\gamma : J \to M$ such that $0 \in J$ and $\gamma(0) = p$. Define an equivalence relation on $\mathcal{C}_p$ by saying that $\gamma_1 \sim \gamma_2$ if $(f \circ \gamma_1)'(0) = (f \circ \gamma_2)'(0)$ for every smooth real-valued function f defined in a neighborhood of p, and let $\mathcal{V}_p$ denote the set of equivalence classes. Show that the map $\Phi : \mathcal{V}_p \to T_pM$ defined by $\Phi[\gamma] = \gamma'(0)$ is well-defined and yields a one-to-one correspondence between $\mathcal{V}_p$ and T_pM.