MAT 320. HW due Oct 24, 2018

Do problems 17.12, 17.14, 18.8, 18.9 from the textbook.

Problem 1. Let $f(x) = \max(x, |x| - 1)$ and $g(x) = \text{sgn}(|x| - 1)$, where sgn denotes the sign function: $\text{sgn}(x) = 1$ for any $x > 0$, $\text{sgn}(x) = -1$ for any $x < 0$, and $\text{sgn}(0) = 0$. For each of the functions $f, g, f \circ g, g \circ f$ determine the set of all $x \in \mathbb{R}$ at which it is continuous.

Problem 2. Suppose f is a continuous function $[0, 1] \to [0, 1]$ such that $f(0) = 0$ and $f(1) = 1$. Suppose that for any $x \in [0, 1]$, one has $f \circ f \circ f(x) = x$. Prove that for any $x \in [0, 1]$, $f(x) = x$.

Problem 3. For this problem, assume all the properties of $\mathbb{R}^2$, and of the area of sets in $\mathbb{R}^2$, that you want — in particular assume that all sets you encounter below have a well-defined area (MAT 324 is largely devoted to understanding what happens more generally).

Let $S \subset \mathbb{R}^2$ be a subset of the plane contained in the unit disk around the origin. Show that there exists a line through the origin that divides S into two sets of equal area.

Problem 4. Let S be the set of all real numbers $0 < x < 1$ such that the decimal expansion of x consists only of digits 5 and 7. Does there exist an unbounded continuous function on S?