MAT 320. HW due Oct 17, 2018

Do problems 12.12, 14.10, 14.14, 15.7 from the textbook.

Problem 1. Suppose (s_n) and (t_n) are bounded sequences. Define the sequence (a_n) by $a_n := \max(s_n, t_n)$, for all n. Prove

$$\limsup a_n \leq \max(\limsup s_n, \limsup t_n).$$

Problem 2. Determine which of the following series converge (with proofs): a) $\sum(-1)^n n^{-1/3}$; b) $\sum \frac{n^2+1}{2^n}$; c) $\sum \frac{(2^n)!}{2^{(2^n)}}$.

Problem 3. Does there exist a real number $0 < x < 1$, such that the decimal expansions of x and x^2 are the same, starting from the millionth term, and neither expansion has an infinite tail of zeroes?

Problem 4. Ancient Babylonians (and others before them — look it up) used base 60 computations. This means the number 60 for them played the same role as 10 plays for us, so you can imagine they had 60 different "digits" 0,1,...,59. This system is called sexagesimal. Sketch the definition of a sexagesimal expansion of a real number, and determine rigorously which real numbers have sexagesimal expansions that end in an infinite tail of zeroes.