MAT 320. HW 12, due Dec 5, 2018

Do problems 23.5, 23.7, 24.3, 24.6, 24.15 from the textbook.

Problem 1. Prove that if a sequence of functions $f_n : X \to \mathbb{R}$ converge uniformly to a function $f : X \to \mathbb{R}$, and if $g : X \to \mathbb{R}$ is a bounded function, then the sequence of functions $g \cdot f_n$ converge uniformly to $g \cdot f$. Construct a counterexample to this if g is unbounded.

Problem 2. Suppose $f_n : [0, 1] \to \mathbb{R}$ is a sequence of continuous functions that converges uniformly on $(0, 1)$. Prove that it converges uniformly on $[0, 1]$.

Problem 3. State the most general version you can of the previous problem that holds for maps of arbitrary metric spaces.