

Syllabus

Lectures: Tuesday and Thursday, 11:00 a.m.–12:20 p.m., Physics P130.

Office hours: Tuesday, 2:15–3:15 p.m., Math Tower 4-121, and by appointment.

Email: radu.laza@stonybrook.edu

Course description. This course is an introduction to the theory of smooth complex algebraic surfaces. We will develop the birational and cohomological tools needed to study surfaces, including intersection theory, the Riemann–Roch theorem, blow-ups, ruled and rational surfaces, minimal models, and Kodaira dimension. Important examples will include Hirzebruch surfaces, del Pezzo surfaces, K3 surfaces, and elliptic surfaces. We will also discuss linear systems on K3 surfaces and the role of du Val singularities in the birational geometry of surfaces. The course will culminate in the Enriques–Kodaira classification. The main reference is Beauville’s textbook [B], supplemented by selected material from the additional references below.

Prerequisites. A first graduate course in algebraic geometry, including divisors, line bundles, sheaves, and sheaf cohomology. Familiarity with basic complex geometry is useful but not required.

Primary reference.

[B] A. Beauville, *Complex Algebraic Surfaces*, 2nd ed., London Mathematical Society Student Texts 34, Cambridge University Press, 1996.

Additional references.

[BHPV] W. Barth, K. Hulek, C. Peters, and A. Van de Ven, *Compact Complex Surfaces*, 2nd ed., Springer, 2004.

[F] R. Friedman, *Algebraic Surfaces and Holomorphic Vector Bundles*, Springer, 1998.

[H] D. Huybrechts, *Lectures on K3 Surfaces*, Cambridge University Press, 2016.

Tentative Weekly Schedule

Week 1 (Aug. 25, 27). Complex surfaces and their numerical invariants ([B], Ch. I).

Week 2 (Sept. 1, 3). Intersection theory, adjunction, and the Riemann–Roch theorem ([B], Ch. I).

Week 3 (Sept. 8, 10). Birational maps, blow-ups, and exceptional curves ([B], Ch. II).

Week 4 (Sept. 15, 17). Minimal models and the factorization of birational maps ([B], Ch. II).

Week 5 (Sept. 22, 24). Ruled surfaces and geometrically ruled surfaces ([B], Ch. III).

Week 6 (Sept. 29, Oct. 1). Rational surfaces and Castelnuovo’s rationality criterion ([B], Chs. IV–V).

Week 7 (Oct. 6, 8). Complements and review of rational and ruled surfaces; surfaces with $p_g = 0$ and $q \geq 1$ ([B], Chs. V–VI).

Week 8 (Oct. 13, 15). No class: Fall Break (Oct. 13); instructor away (Oct. 15).

Week 9 (Oct. 20, 22). Pluricanonical systems, Kodaira dimension, and the classification strategy ([B], Ch. VII).

Week 10 (Oct. 27, 29). Surfaces of Kodaira dimension 0; overview of the four classes ([B], Ch. VIII).

Week 11 (Nov. 3, 5). K3 and Enriques surfaces; basic examples and constructions ([B], Ch. VIII).

Week 12 (Nov. 10, 12). Abelian and bielliptic surfaces; elliptic fibrations ([B], Chs. VIII–IX).

Week 13 (Nov. 17, 19). Surfaces of Kodaira dimension 1 ([B], Ch. IX).

Week 14 (Nov. 24, 26). Enriques–Kodaira classification summary ([B], Chs. VII–IX); no class Nov. 26 (Thanksgiving).

Week 15 (Dec. 1, 3). Surfaces of general type; examples, geography, and concluding perspectives ([B], Ch. X).

The schedule is tentative and may be adjusted according to the pace and interests of the class.