Name:

Math 122 (Fall '12)

Midterm 1

October 4, 2012

Note: There are 6 questions. The first 2 questions are considered to be easy. Questions 3,4,5 have medium difficulty. Question 6 is a True/False question of medium difficulty. Make sure that you attempt to solve all questions, as there will be partial credit.

1. (15pts)	
2. (15pts)	
3. (20pts)	
4. (20pts)	
5. (20pts)	
6. (10pts)	
Total (100pts)	

1. (15pts) Solve the following equations:

(1) $x^2 + 2x + 3 = x^2 + 5$

(2) $(x + 1)^2 = 4x^2$

(3) $3^x = 5 \cdot 2^x$

(4) $7x^{-\frac{3}{2}} = x^5\sqrt{x}$

(5) $\ln(5x) = \ln(2x^2) + 3$

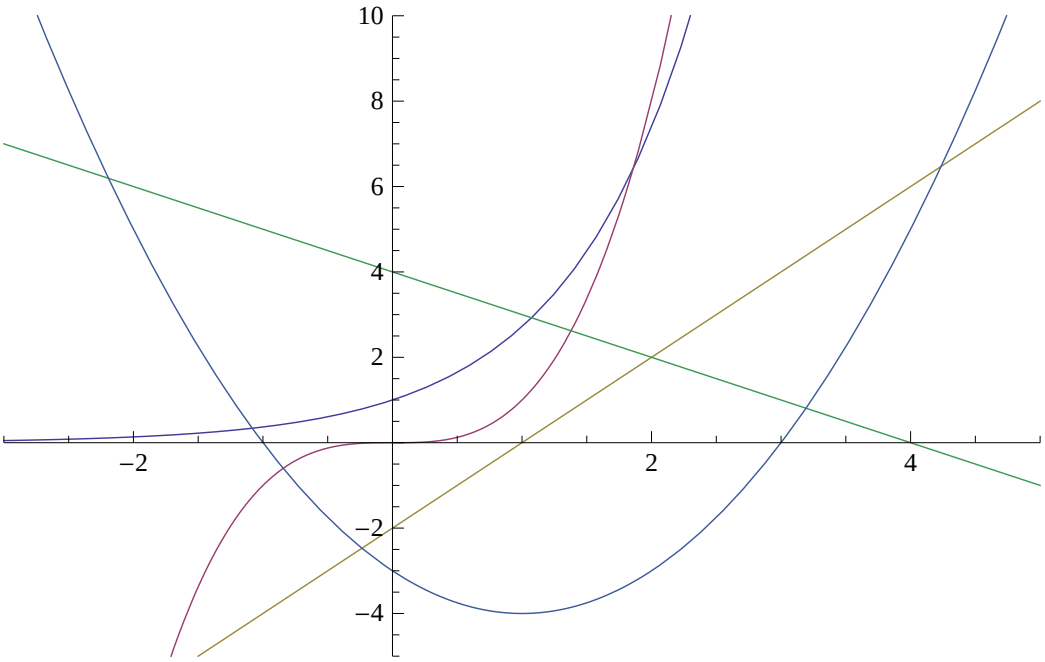
2. (15pts) You are given 8 functions and 5 graphs. On the graph, match the functions with their graphs.

(A) e^x (D) x^3 (G) $4 - x$

(B) $\ln x$ (E) $x^2 - 2x - 3$ (I) $2x - 2$

(C) x^2 (F) $4 + x$

(Of course, three of the given functions are not graphed.)



3. (20pts) You need \$10,000 dollars in your account 5 years from now. Assume

(S) The account is a savings account with an interest rate is 4% a year, compounded continuously. You only make an initial deposit D (no further contribution).

(C) The account is a checking account and there is no interest. You make an initial deposit D and then additional contributions of \$1000 each year.

For each of the two cases (S) and (C):

i) Write down a formula modeling the balance of the account after t years.

(S):

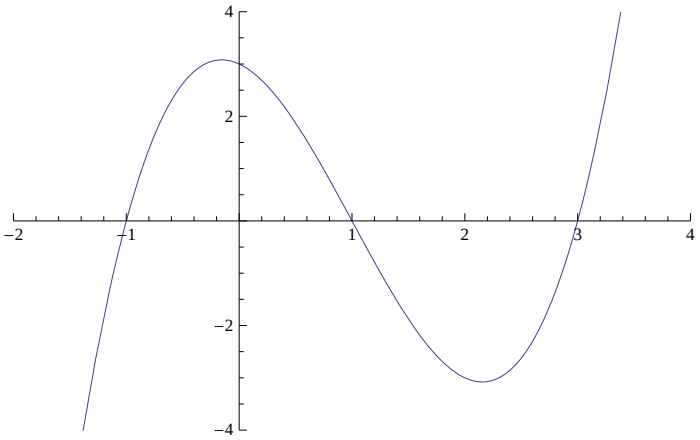
(C):

ii) What is the size of the initial deposit D necessary necessary to achieve your goal.

(S):

(C):

4. (20pts) The following is the graph of **the first derivative** $f'(x)$:



i) Find the intervals on which $f(x)$ is increasing and those on which $f(x)$ is decreasing.

ii) Find the intervals on which $f(x)$ is concave up and those on which $f(x)$ is concave down.

iii) Assume $f(0) = 1$. Which of the following is possible: $f(1) = 3$ or $f(1) = 0$. Explain.

iv) Sketch a graph of $f''(x)$.

v) Assume $f(0) = 1$. Sketch a graph of $f(x)$.

5. (20pts) This exercise tests your understanding of using derivatives to approximate the values of functions. The first two items are standard questions. The last two are slightly more challenging.

(1) Assume that $f(20) = 10$, $f'(20) = 0.3$. What is a reasonable value for $f(22)$?

(2) Approximate $\ln(1.5)$. Hint: $(\ln(x))' = \frac{1}{x}$.

(3)* Illustrate the approximation from (2) on a graph. (Hint: The tangent to the graph at a is an approximation of the graph of f near a. Thus, $f(x) \approx f(a) + f'(a)(x - a)$.)

(4)* Now use the fact that $\ln(x)$ is a concave down function and the graph from (3), to decide if the approximation from (2) is an under-estimate (you get a smaller value than $\ln(1.5)$) or an over-estimate (you get a larger value than $\ln(1.5)$). Explain.

6. (10pts) True/False or **Fill-in**

(1) The graph of the linear function $m(x) = 3x + 2$ has slope 2.

(2) $\ln(a + b) = \ln(a) + \ln(b)$.

(3) The relative rate of change for $g(t) = (1 + r)^t$ is _____.

(4) The function $f(t) = 5^t$ grows more quickly than the function $g(t) = e^t$.

(5) The average rate of change of a function is the slope of the line between two points on the graph of the function.

(6) The instantaneous rate of change $f'(a)$ of a function f is the slope of the _____ line at the point _____.

(7) The instantaneous rate of change for e^x is 1.

(8) If $f'(100) = 2$ and the derivative of f is not changing rapidly, then $f(102) - f(100) \approx 4$.

(9) If $f'' < 0$ on an interval, then f is decreasing on that interval.

(10) If $f(x)$ is concave up, then _____ is increasing.