Name:

Math 122 (Fall '12)

Final Exam

December 12, 2012

Time: 11:15-1:45

1. (25pts)		5. (25pts)	
2. (25pts)		6. (25pts)	
3. (25pts)		7. (25pts)	
4. (25pts)		8. (25pts)	
Total I (100pts)		Total II (100pts)	

Total Score:

Basic Rules for the final exam:

(1) Make sure that there is an empty seat to your right and left. Otherwise, the proctor will mark your exam and I will impose a heavy penalty (up to voiding the exam).

(2) Make sure that you sign your exam. Make sure that you sign the photo roster when turning in the exam.

(3) You are allowed 3 index cards (size 3x5), but nothing larger (i.e. you can not use a larger page as substitute for the 3 index cards).

(4) **NO Calculator** is allowed.

1. (25pts) Solve the following equations:

(1) $2x + 5 = 3(x - 3)$

(2) $(2x - 1)^2 = 3x - 2$

(3) $e^{2x} = 2 \cdot e^x$

(4) $\ln x + \ln 2x = \ln 3x$

(5) $\sqrt{4x} = x^{\frac{5}{2}}$

2. (25pts) Find the derivatives for each of the following functions:

(1) $3e^x - 2x^e$

(2) $x^3 \ln x + 1$

(3) $2xe^{x^2}$

(4) $\sqrt{e^x + e^{-x}}$

(5) $\frac{x^2-1}{x^2+1}$ (Note: full simplification required in this case)

3. (25pts) Find an antiderivative for each of the following functions

(1) $x^3 + x^{-3}$

(2) $2\sqrt{x} - \frac{3}{x}$

(3) $e^x\sqrt{e^x + 1}$

(4) $x^2(2x^3 + 1)^{100}$

(5) $\frac{e^x \ln(e^x+1)}{e^x+1}$

4. (25pts) [Tests your understanding of the Fundamental Thm. of Calculus]
A. Compute the following definite integrals

(1) $\int_1^4 \sqrt{x}\, dx$

(2) $\int_0^1 2xe^{x^2}\, dx$

B. Say whether each of the following formulas is true or false

(3) $\int x \ln x\, dx = \frac{1}{2}x^2 \ln x - \frac{x^2}{4} + C$

(4) $\int \frac{x}{x^2+1}\, dx = \frac{1}{2}\ln(x^2+1) + C$

C. Assume $f(x)$ is the derivative of the function $F(x) = xe^2$. Compute the integral $\int f(x)\, dx$.

5. (25pts)
A. Let $P(t)$ represent the price of a share of stock of a corporation at time t. What does each of the following statements tell us about the signs of the first and second derivatives of $P(t)$?

i) "The price of the stock is rising faster and faster".

ii) "The price of the stock is close to bottoming out".

B. An old rowboat has sprung a leak. Water is flowing into the boat at a rate, $r(t)$, given in the following table

t minutes	0	5	10	15
$r(t)$ liters/min	12	20	24	26

i) Give a lower estimate for the volume of water that has flowed into the boat during the 15 minutes.

ii) Give an upper estimate for the volume of water that has flowed into the boat during the 15 minutes.

iii) Draw a graph to illustrate the lower estimate.

6. (25pts)

(A) Find the area under $y = x^3 + 2$ between $x = 0$ and $x = 2$. Sketch this area.

(B) Find the area enclosed by $y = 3x$ and $y = x^2$.

(C) The equation of the circle of center $(3, 0)$ and radius 2 is $y^2 + (x-2)^2 = 9$, which is the same as saying $y = \pm\sqrt{9 - (x-2)^2}$. Use this information to compute

$$\int_1^3 \sqrt{9 - (x-2)^2}\, dx.$$

7. (25pts) Consider the function $f(x) = 3x^4 + 4x^3 - 12x^2 + 1$ on the interval $-3 \le x \le 3$.

(i) Compute $f'(x)$ and $f''(x)$.

(ii) Find all the critical points of f and decide which are local min/max.

(iii) Find the global maximum and minimum of f.

(iv) Find all the inflection points of f. (Hint: use the approximation $\sqrt{7} \cong 2.6$ in computations)

(v) Graph the function f (on the given interval)

8. (25pts) True/False/Fill-in

(1) A linear function has a constant rate of change.

(2) An exponential function has a constant rate of change.

(3) The slope of the secant line between two points $(a, f(a))$ and $(b, f(b))$ gives the average rate of change for the function $f(x)$ on the interval $[a, b]$.

(4) The quantity $f'(a)$ represents the slope of the ________ line at the point ________.

(5) A critical point for a function is always a point of local maximum or minimum.

(6) The accumulated change for $f(x)$ on the interval $[a, b]$ is measured by the quantity ________.

(7) A 4-term Riemann sum on the interval $4 \leq t \leq 6$ has $\Delta t = 2$.

(8) If the graph of $f(x)$ has more area below the x-axis than above the x-axis when $1 \leq x \leq 10$ then $\int_1^{10} f(x)\, dx > 0$.

(9) Assume $f(x) = F'(x)$, then $\int_a^b f(x)\, dx =$________.

(10) If $\int_0^5 F'(t)\, dt < 0$, then $F(0) > F(5)$.