

## MAT126.R01: QUIZ 9

### SOLUTIONS

Find the volume of the solid obtained by rotating the region bounded by the curves

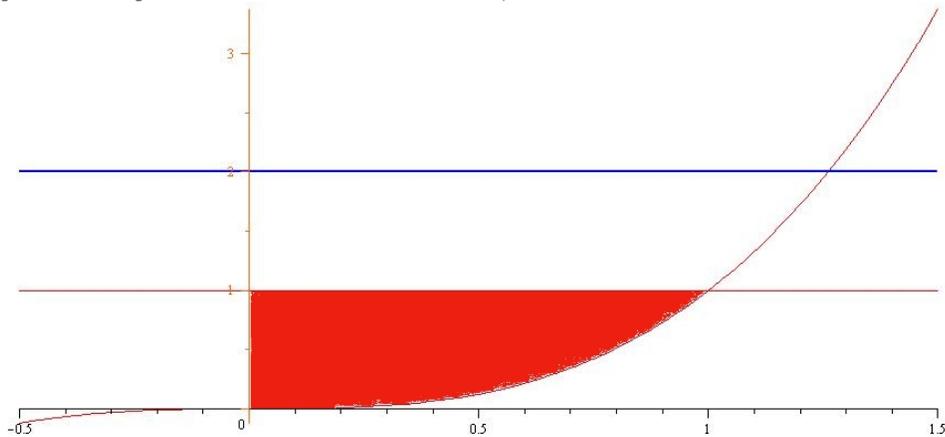
$$y = x^3, \quad x = 0, \quad y = 1$$

about  $y = 2$ .

(Sketch the region first.)

*Intersection points:  $x = 0$  and  $y = x^3$  intersect at the origin.*

*$y = 1$  and  $y = x^3$  intersect when  $x^3 = 1$ , i.e. at  $x = 1$ .*



*Since the region is “below” the axis of revolution, the inner shell is formed by  $y = 1$  and the outer shell by  $y = x^3$ .*

$$\begin{aligned} \text{The volume is } & \int_0^1 \pi ((x^3 - 2)^2 - (1 - 2)^2) dx = \pi \int_0^1 x^6 - 2x^3 + 4 - 1 dx = \\ & \pi \left( \frac{x^7}{7} - 2\frac{x^4}{4} + 3x \right) \Big|_0^1 = \pi \left( \frac{1}{7} - 2\frac{1}{4} + 3 - 0 \right) = \frac{37\pi}{17} \end{aligned}$$