MAT126.R02: QUIZ 1

SOLUTIONS

1. Find the expression for the area under the graph of

$$f(x) = x^3 + \cos x$$

from $x = 1$ to $x = 3$. Do not evaluate the formula you obtain.

The area is the limit of $R_n = \Delta x(f(x_1) + \cdots + f(x_n)) = \Delta x((x_1^3 + \cos x_1) + \cdots + (x_n^3 + \cos x_n))$

Here $\Delta x = \dfrac{3-1}{n} = \dfrac{2}{n}$ *and* $x_k = 1 + \dfrac{2}{n}k = 1 + \dfrac{2k}{n}$.

Hence the area is $\displaystyle \lim_{n\to\infty} \sum_{k=1}^{n} \frac{2}{n}\left[\left(1 + \frac{2k}{n}\right)^3 + \cos\left(1 + \frac{2k}{n}\right)\right]$.

2. Determine the region whose area is equal to the given expression:

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{3}{n} \ln\left(7 + \frac{3k}{n}\right).$$

Do not evaluate the limit.

Here $\Delta x = \frac{3}{n}$, $x_k = 7 + \dfrac{3k}{n}$, *and* $f(x_k) = \ln\left(7 + \frac{3k}{n}\right)$.

Since $\Delta x = \dfrac{b-a}{n}$ *and* $x_k = a + k\Delta x$, *we see that* $a = 7$ *and* $b - a = 3$. *This makes* $b = 10$.

The function $f(x)$ *is* $\ln x$.

Therefore the region is under the graph of $f(x) = \ln x$ *from* $x = 3$ *to* $x = 10$.