MAT 126: PRACTICE FOR MIDTERM 1

SOLUTIONS

Chapter 4 Review Exercises

51. $f(x) = e^x - 2x^{-1/2}$, hence the antiderivative is $F(x) = e^x - 2\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C = e^x - 4x^{1/2} + C = e^x - 4\sqrt{x} + C$.

52. $g(t) = \dfrac{1+t}{\sqrt{t}} = \dfrac{1}{\sqrt{t}} + \dfrac{t}{\sqrt{t}} = t^{-1/2} + t^{1/2}$, hence the antiderivative is $G(t) = \dfrac{t^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + \dfrac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = 2\sqrt{t} + 2t^{3/2}/3 + C$.

55. $f''(x) = 1 - 6x + 48x^2$, hence $f'(x) = x - 3x^2 + 16x^3 + C$ (the antiderivative of $f''(x)$). Since $f'(0) = 2$, we have $0 - 3(0)^2 + 16(0)^3 + C = 2$, that is $C = 2$. $f(x) = \dfrac{x^2}{2} - x^3 + 4x^4 + 2x + D$. Since $f(0) = 1$, we have $\dfrac{0^2}{2} - 0^3 + 4(0)^4 = 2(0) + D = 1$, that is $D = 1$. Therefore, $f(x) = 1 + 2x + \dfrac{x^2}{2} - x^3 + 4x^4$.

57. $s(t)$ is an antiderivative of $v(t) = 2t - \dfrac{1}{1+t^2}$, that is $s(t) = t^2 - \tan^{-1} t + C$. Since $s(0) = 1$, $0^2 - \tan^{-1} 0 + C = 1$. Thus $C = 1$ and $s(t) = t^2 - \tan^{-1} t + 1$.

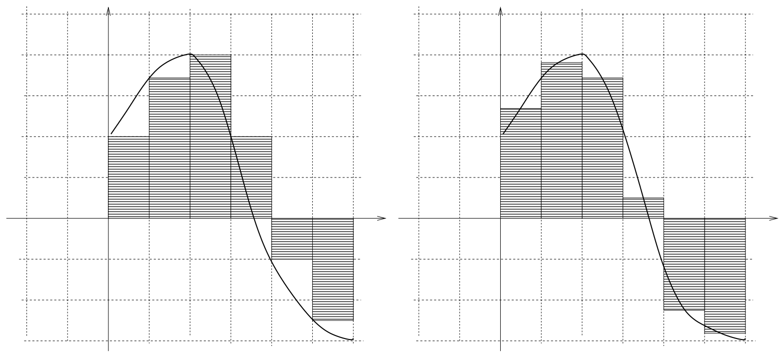
Chapter 5 Review Exercises

1. In both parts $\Delta x = 1$ (the interval $[0, 6]$ is divied into six parts).

(a) Left endpoints: 0, 1, 2, 3, 4, 5, so the sum is
$(f(0)+f(1)+f(2)+f(3)+f(4)+f(5))\Delta x = (2+3.5+4+2-1-2.5)1 = 8$

(b) Midpoints: $0.5, 1.5, 2.5, 3.5, 4.5, 5.5$, so the sum is
$(f(0.5) + f(1.5) + f(2.5) + f(3.5) + f(4.5) + f(5.5))\Delta x = 3 + 3.8 + 3.5 + 0.5 - 2 - 2.9 = 5.9$

The sums represent the total areas of rectangles drawn below ((a) on the left; (b) on the right):



3. $\int_0^1 (x+\sqrt{1-x^2})\,dx = \int_0^1 x\,dx + \int +0^1 \sqrt{1-x^2})\,dx$. The first integral is the area of the triangle formed by the x-axis, the line $y = x$ and the vertical line at $x = 1$, thus it equals $1/2$ (alternatively, you can compute by using anti-derivatives). The second integral of the function $y = \sqrt{1-x^2}$ (i.e. $x^2 + y^2 = 1$ and the graph of the function is the upper semi-circle of radius 1) is the area of the quarter of the circle of radius 1, i.e. $\pi/4$. Answer: $1/2 + \pi/4$.

4. This is the sum of the areas of rectangles of width Δx and height determined by the function $\sin x$. Hence it equals
$\int_0^\pi \sin x\,dx = -\cos x|_0^\pi = -\cos\pi - (-\cos 0) = 2.$

5. $\int_4^6 f(x)\,dx = \int_0^6 f(x)\,dx - \int_0^4 f(x)\,dx = 10 - 7 = 3.$

9. $\int_1^2 (8x^3+3x^2)\,dx = 8\frac{x^4}{4} + 3\frac{x^3}{3}\Big|_1^2 = 2x^4 + x^3|_1^2 = 2\cdot 2^4 + 2^3 - (2\cdot 1 + 1) = 37$

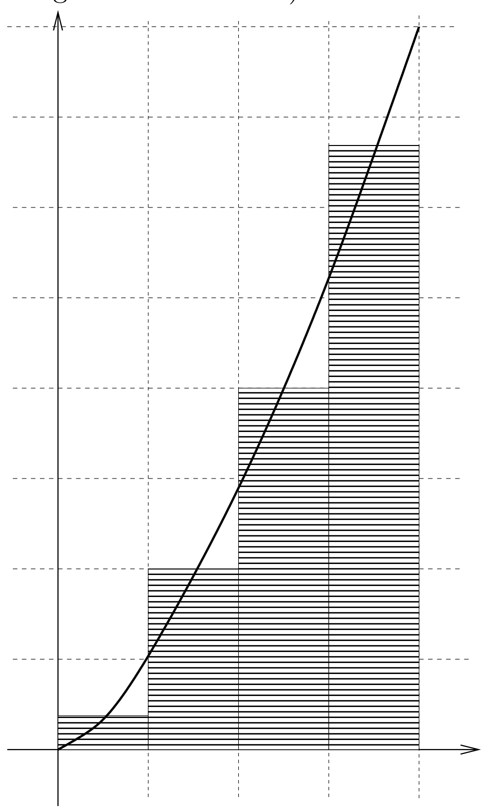
13. $\int \left(\frac{1-x}{x}\right)^2 dx = \int \frac{1-2x+x^2}{x^2}\,dx = \int \frac{1}{x^2} - 2\frac{1}{x} + 1\,dx = \int x^{-2} - 2x^{-1} + 1\,dx$
$= -\frac{1}{x} - 2\ln x + x + C$

14. $\int_0^1 (\sqrt[4]{u}+1)^2\,du = \int_0^1 \sqrt{u} + 2\sqrt[4]{u} + 1\,du = \int_0^1 u^{1/2} + 2u^{1/4} + 1\,du = \frac{u^{3/2}}{3/2} + 2\frac{u^{5/4}}{5/4} + u\Big|_0^1 =$
$\frac{2}{3} + \frac{8}{5} + 1 - (0+0+0) = \frac{49}{15}$

23. $\int_0^5 \frac{x}{x+10}\,dx = \int_0^5 \frac{x+10-10}{x+10}\,dx = \int_0^5 1 - \frac{10}{x+10}\,dx = x - 10\ln(x+10)|_0^5 =$
$5 - 10\ln 15 - (0 - 10\ln 10) = 5 + 10(\ln 10 - \ln 15) = 5 + 10\ln(10/15) =$
$5 + 10\ln(2/3)$,

using $\int \frac{1}{x+10}\,dx = \ln(x+10) + C$ (check by differentiation).

37. Here is the rough estimate: (I basically use midpoints here; the extra bits in the rectangles more or less cancel the area under the graph that rectangles failed to cover)



The rough estimate is $0.3 + 2 + 4 + 6.7 = 13$

The exact area is $\displaystyle\int_0^4 x\sqrt{x}\,dx = \int_0^4 x^{3/2}\,dx = \left.\frac{x^{5/2}}{5/2}\right|_0^4 = \frac{4^{5/2}}{5/2} - 0 = \frac{64}{5}$

39. $F'(x) = \dfrac{x^2}{1+x^3}$

40. Let $F(x) = \displaystyle\int_1^x \frac{1-t^2}{1+t^4}\,dt$. Then $F'(x) = \dfrac{1-x^2}{1+x^4}$

$g(x) = F(\sin x)$. Then $g'(x) = (F(\sin x))' = F'(\sin x)(\sin x)' = \dfrac{1-\sin^2 x}{1+\sin^4 x}\cos x = \dfrac{\cos^2 x}{1+\sin^4 x}\cos x = \dfrac{\cos^3 x}{1+\sin^4 x}$

41. $y = \displaystyle\int_{\sqrt{x}}^x \frac{e^t}{t}\,dt = \int_{\sqrt{x}}^0 \frac{e^t}{t}\,dt + \int_0^x \frac{e^t}{t}\,dt = -\int_0^{\sqrt{x}} \frac{e^t}{t}\,dt + \int_0^x \frac{e^t}{t}\,dt$

Let $F(x) = \displaystyle\int_0^x \frac{e^t}{t}\,dt$. Then $F'(x) = \dfrac{e^x}{x}$.

$\displaystyle\int_0^{\sqrt{x}} \frac{e^t}{t}\,dt = F(\sqrt{x})$. $(F(\sqrt{x}))' = F'(\sqrt{x})(\sqrt{x})' = \dfrac{e^{\sqrt{x}}}{\sqrt{x}}\dfrac{1}{2\sqrt{x}} = \dfrac{e^{\sqrt{x}}}{2x}$.

Thus $y' = -\dfrac{e^{\sqrt{x}}}{2x} + \dfrac{e^x}{x} = \dfrac{2e^x - e^{\sqrt{x}}}{2x}$

63. (a) Displacement: $\displaystyle\int_0^5 t^2 - t\,dt = \frac{t^3}{3} - \frac{t^2}{2}\Big|_0^5 = \frac{5^3}{3} - \frac{5^2}{2} - 0 = \frac{175}{6}$ m.

(b) Distance: $\displaystyle\int_0^5 |t^2 - t|\,dt$. To compute this integral, we need to figure out where $t^2 - t$ is positive and negative:

$t^2 - t = 0$ at $t = 0, 1$. Hence, $t^2 - t < 0$ for $0 < t < 1$ and $t^2 - t > 0$ on $t > 1$ (we don't care about what happens for $t < 0$ because the integral is computed over $[0, 5]$.)

$\displaystyle\int_0^5 |t^2 - t|\,dt = \int_0^1 -(t^2 - t)\,dt + \int_1^5 |t^2 - t|\,dt = -\frac{t^3}{3} + \frac{t^2}{2}\Big|_0^1 + \frac{t^3}{3} - \frac{t^2}{2}\Big|_1^5 =$
$\displaystyle -\frac{1}{3} + \frac{1}{2} + \frac{5^3}{3} - \frac{5^2}{2} - \left(\frac{1}{3} - \frac{1}{2}\right) = \frac{177}{6} = 29.5$ m.

65. $\displaystyle\int_0^8 r(t)\,dt = R(8) - R(0)$, where $R(t)$ is an antiderivative of $r(t)$, i.e. the consuption of oil from $t = 0$ (the year 2000) to year t. The integral represents the oil consumption between the years 2000 and 2008 measured in barrels.