# AMS 102: FINAL EXAM PRACTICE

SOLUTIONS

## Chapter 1

**1.23.** (a) False: $\alpha$ and $\beta$ are not related directly.

(b) False: we may fail to reject the null hypothesis when it's true. (That is, we may fail to reject the null hypothesis and commit no error.)

(c) True: the rejection region should included only ages greater than 76.

**1.30.** (a) To the left. The plot for Machine B is concentrated more to the left than the plot for Machine A.

(b) Type I error occurs if we decide to reject $H_0$ (i.e. go with Machine B), when we actually deal with Machine A. This means that we deal with Machine A and the number of flaws is 2 or 1 (more extreme). The probability of this $\alpha = (2+1)/15 = 0.2$.

Type II error occurs if we fail reject $H_0$ (i.e. go with Machine A), when we actually deal with Machine B. This means that we deal with Machine B but the number of flaws is more than 2. The probability of this is $\beta = (3+2+1)/15 = 0.4$.

(c) Under $H_0$ the probability of getting 4 flaws is 4/15. This is the $p$-value.

(d) Since 4/15 is greater than $\alpha = 0.2$, the data is not statistically significant.

**1.35.** (a) New-model bulbs.

(b) $H_0$: the average lifetime is 40 hours.

$H_1$: the average lifetime is greater than 40 hours.

(c) 10%.

(d) Any value between 0 and $\alpha$ (i.e. 0.10).

(e) If the data is statistically significant for $\alpha = 0.10$, then $p < 0.10$. Therefore, $p < 0.15$ and the data will remain statistically significant for $\alpha = 0.15$.

However, $p$ can be less or greater than 0.05, so we can't say anything about $\alpha = 0.05$. The data may or may not be statistically significant.

## Chapter 9

**9.41.** (a) $H_0 : p = 0.72$, $H_1 : p > 0.72$. The direction of the extreme is to the right.

(b) $H_0 : p = 0.90$, $H_1 : p < 0.90$. The direction of the extreme is to the left.

(c) $H_0 : p = 0.60$, $H_1 : p \neq 0.60$. The direction of the extreme is two-sided.

**9.49.** (a) Here $p$ is the proportion of workers in the workforce who expect to leave their job in the next five years. $H_0 : p = 0.5$, $H_1 : p > 0.5$.

(b) Test statistic: $\hat{p} = \frac{380}{714} =$. Under $H_0$ we expect $\hat{p}$ to be distributed as $N\left(0.5, \sqrt{\dfrac{0.5(1-0.5)}{714}}\right)$. Thus $z = (\hat{p} - 0.5)/\sqrt{\frac{0.5(1-0.5)}{714}} \approx 1.72$. The $p$-value is $1 - 0.9573 = 0.0427$.

(c) Since $p > 0.05$, the results are statistically significant at the 5% level.

**9.51.** (a) $H_0$: $p = 0.6$; $H_1$: $p > 0.6$.

(b) The sample proportion is $\hat{p} = 0.65$. This is distributed as $N\left(0.6, \sqrt{\dfrac{0.6(1-0.6)}{200}}\right) \approx N(0.6, 0.035)$. Thus the $z$-test value is $\dfrac{0.65-0.6}{0.035} \approx 1.429$. The $p$-value is $P(z > 1.429) \approx 1 - 0.9236 = 0.0764$. The $p$-value is greater than the significance level of 0.01, hence is not statistically significant.

(c) No. For a large sample size, the sampling distribution of proportion will be approximately normal, regardless of how the population is distributed.

**9.53.** (a) $\hat{p} = 0.48$. $SE(\hat{p}) = \sqrt{\dfrac{0.48(1-0.48)}{450}} \approx 0.0236$. For 90% confidence, $z^* = 1.645$, hence the confidence interval is $0.48 \pm 1.645 \cdot 0.0236 \approx 0.48 \pm 0.0388$ or $(0.4412, 0.5188)$.

(b) $E = 1.645 \cdot 0.0236 \approx 0.0388$

(c) The margin of error would increase. Specifically, for 99% confidence, $z^* = 2.576$, thus $E = 2.576 \cdot 0.0236 \approx 0.0608$

**9.55.** (a) Observational study.

(b) No: the sample is not random because alcohol levels were recorded only for dead bicyclists (which obviously do not represent all bicyclists on the road). Therefore, the sample can't be used for making any conclusions about the general population, whatever the confidence level.

(c) No. We could only do so if we concluded that the alcohol blood level of those who suffered a major accident was significantly higher than in the general population. Such data is unavailable. (For all we know, bicyclists who do not get into accidents may have higher blood levels on average. This would mean that drinking prevents you from getting into accident.)

## Chapter 10

**10.39.** (a) $H_0$: The mean germination is not faster than 17.1 days

$H_1$: The mean germination is faster than 17.1 days

(b) We need to use the $t$-test. Sample mean $\bar{x} = (15 + 17 + 12 + 11 + 14 + 18 + 12 + 20 + 15 + 16)/10 = 15$. Sample standard deviation

$$s = \sqrt{\frac{(15-15)^2+(17-15)^2+(12-15)^2+(11-15)^2+(14-15)^2+(18-15)^2+(12-15)^2+(20-15)^2+(15-15)^2+(16-15)^2}{9}} \approx 2.8674.$$

Thus $t = \dfrac{15-17.1}{2.8674/\sqrt{10}} \approx -2.316$. From the table with $df = 10 - 1 = 9$, $p$-value falls between 0.025 and 0.01

(c) We reject $H_0$ because the $p$-value is significant (less than 0.10). Thus with 90% confidence we conclude that the average mean germination is faster than 17.1 days.

**10.41.** (a) $P(x < 285) = P\left(z < \frac{285-270}{10}\right) = P(z < 1.5) = 0.9332$.

(b) (i) The probability that at least one lasts longer than 285 days is $1-$the probability that none of them lasts longer than 285 days. The probability that $X_1$, $X_2$, $X_3$, $X_4$ are less than 285 is $0.9332^4 \approx 0.7584$. Hence the answer is $1 - 0.7584 = 0.2416$.

(ii) The sample mean of four pregnancies is distributed as $N\left(270, \frac{10}{\sqrt{4}}\right) = N(270, 5)$. $P(\bar{X} < 285) = P\left(z < \frac{285-270}{5}\right) = P(z < 3) = 0.9987$.

(c) $H_0$: $\mu = 270$; $H_1$: $\mu < 270$.

(d) The test statistic is $t = \dfrac{265.8 - 270}{12.2/\sqrt{15}} \approx -1.333$. The $p$-value is $P(t < -1.333) = P(t > 1.333)$ is slighlty higher than 0.10 (from the chart for $df = 15 - 1 = 14$).

(e) The $p$-value is higher than 0.05, hence we fail to reject $H_0$.

(f) We cannot conclude that the mean length of pregnancy for women working outside the home is shorter than 270 days.

**10.47.** (a) Apartments in the suburb.

(b) Living area of the apartment.

(c) 1325 sq. feet.

(d) $1325 \pm t^* \dfrac{42}{\sqrt{100}} \approx 1325 \pm 1.986 \cdot 4.2$ or $(1316.6, 1333.3)$.

(e) No. The true value either lies in the interval or it doesn't. 95% confidence means that 95% of the samples produce intervals that contain the true mean.

(f) We could increase the confidence level (i.e. increase $t^*$) or take a smaller sample (i.e. make $\dfrac{s}{\sqrt{n}}$ larger).

**10.49.** (a) The confidence interval is $11.5 \pm z^* \dfrac{0.2}{\sqrt{25}} = 11.5 \pm 2.576 \cdot 0.04 \approx 11.5 \pm 0.103$ or $(11.397, 11.603)$.

(b) $E = 2.576 \cdot 0.04 \approx 0.103$.