AMS 102: MIDTERM II PRACTICE

SOLUTIONS

Chapter 1

1.4. H_0: at most 40% of whales leave the area; H_1: more than 40% leave the area.

1.6. (a) Type I: assume the electricity is off when it's on (and then touch the wires...).

(b) Type I: assume the brakes are operational when they are not (and then try to brake).

(c) Type I: assume the snake is not poisonous when it is (and then get bitten). On the other hand, if I am looking for snake venom, then Type II is worse because I'll try to get venom from a snake that doesn't have it.

(d) Type II: assume it's safe to cross the street when it's not (and then have a car from part (b) coming my way).

1.12. (a) H_0: Cephaline works as good or better than Septaphine. H_1: Septaphine works better.

(b) H_1 was supported. A possible mistake is Type I: rejecting H_0 when it's true, i.e. assuming that Septaphine works better when it doesn't.

1.47. (c) Sometimes yes, and sometimes no. To be statistically significant at the 5% level, the data must have a p-value less than 0.05. But such a p-value can be either greater than 0.01 (not significant at the 1% level) or less than 0.01.

Chapter 7

7.71. (a) 20% of 70%: $0.2 \cdot 0.7 = 0.14$, hence 14%.

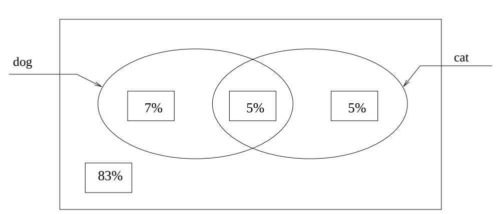
(b) 15% of 14%: $0.15 \cdot 0.14 = 0.021$, hence 2.1%.

(c) Brown-eyed and red-haired: 14% (from (a))

Non-brown-eyed and red-haired: 40% of 30%: $0.4 \cdot 0.3 = 0.12$, hence 12%.

Red-haired total: $14\% + 12\% = 26\%$.

7.73. To construct the Venn diagram, note first that $100 - 17 = 83\%$ own neither the dog nor the cat. If even A is "owns a dog" and event B is "owns a cat", then $P(A) = 12\%$, $P(B) = 10\%$, $P(A \text{ or } B) = 17\%$. Hence $P(A \text{ and } B) = P(A) + P(B) - P(\text{A or B}) = 5\%$.



(a) 5%.
(b) $\dfrac{\text{P(owns a dog but not a cat)}}{\text{P(doesn't own a cat)}} = \dfrac{7}{7+83} \approx 0.778$ (the book's answer is wrong).

7.77. (a) $\dfrac{27+15+18}{100} = 0.6$.
(b) $\dfrac{18+17}{100} = 0.35$.
(c) $\dfrac{18}{100} = 0.18$.
(d) $\dfrac{18}{27+15+18} = 0.15$.
(e) $\dfrac{15+10+18+17}{100} = 0.6$.
(f) Independent: P(democrat)P(opposed)=P(democrat and opposed). We have that the events are independent if $0.6 \cdot 0.35 = 0.18$. But this equality is wrong, so the events are not independent.

7.81. (a) The events would be independent if P(father attended college)P(son attended college)=P(father and son attended college). So, the events are independent if $\dfrac{18+7}{80}\dfrac{18+22}{80} = \dfrac{18}{80}$. This equality is wrong, so the events are not independent.
(b) No: the intersection (the event "both father and son attended college") is not empty.

7.89. $0.5 \cdot 4\% + 0.3 \cdot 6\% + 0.2 \cdot 2\% = 4.2\%$ or 0.042.

7.93. (a) $P(X=4) = 1 - (0.15 + 0.3 + 0.35 + 0.15) = 0.05$.
(b) $0.3 + 0.35 + 0.15 + 0.05 = 0.85$
(c) $\dfrac{\text{P(four toppings)}}{\text{more that two toppings}} = \dfrac{0.05}{0.15+0.05} = 0.25$.

7.95. (a) $P(X=0)+P(X=1)+P(X=2)+P(X=3) = 0.1+0.15+0.3+0.25 = 0.8$.
(b) $0 \cdot 0.10 + 1 \cdot 0.15 + 2 \cdot 0.3 + 3 \cdot 0.25 + 4 \cdot 0.1 + 5 \cdot 0.1 = 2.4$ problems.

7.103. (a) Probability of success in one trial is 0.8, failure $1 - 0.8 = 0.2$
P(9 successes out of 10)$\big(\overset{=}{_{109(0.8)^{9}(0.2)^{1}\approx 0.27}}\big)$
P(10 successes out of 10)$\big(\overset{=}{_{1010(0.8)^{10}(0.2)^{0}=(0.8)^{10}\approx 0.11}}\big)$.
P(9 successes out of 10)+P(10 successes out of 10)$= 0.27 + 0.11 = 0.38$.
(b) Binomial distribution should be approximated by a normal distribution here. Use normal distribution with a mean $100 \cdot 0.8 = 80$ and a standard deviation $\sqrt{100 \cdot 0.8 \cdot 0.2} = 4$. 90% of 100 is 90, so we need to find the probability that the number of successes is greather than 90. $P(x > 90) = P\left(z > \dfrac{90-80}{4}\right) = P(z > 2.5) = 1 - P(z < 2.5) = 1 - 0.9938 = 0.0062$.

Chapter 8

8.33. (a) The distribution is approximately normal (bell-shaped), with the mean equal to the true proportion, 0.2. Thus, the answer is B.

(b) No, the variability is $\sqrt{\dfrac{p(1-p)}{n}}$ and does not depend on the total number of bearings (only on the number of bearings in the sample).

8.35. This is a sampling distribution of proportion with $p = 0.6$, $n = 100$. The distribution is approximately normal, $N\left(p, \sqrt{\dfrac{p(1-p)}{n}}\right) \approx N(0.6, 0.05)$. We need to find $P(\hat{p} < 0.5)$. The z-score for 0.5 is $\dfrac{0.5-0.6}{0.05} = -2$. $P(z < -2) = 0.0228$. (You may have a slightly different answer depending on what approximation you take for $\sqrt{\frac{p(1-p)}{n}}$.)

8.36. The sample proportion is approximately normally distributed with $N\left(0.42, \sqrt{\dfrac{0.42(1-0.42)}{150}}\right) \approx N(0.42, 0.04)$. $P(0.35 < \hat{p} < 0.5) = P\left(\dfrac{0.35-0.42}{0.04} < z < \dfrac{0.5-0.42}{0.04}\right) = P(-1.75 < z < 2) = 0.9772 - 0.0401 = 0.9371$.

8.43. (a) The distribution is $N(173, 30)$. $P(x > 180) = P\left(z > \dfrac{180-173}{30}\right) \approx P(z > 0.23) = 1 - P(z < 0.23) \approx 1 - 0.5910 = 0.4090$.

(b) The distribtuions is $N\left(173, \dfrac{30}{\sqrt{36}}\right) = N(173, 5)$. Thus $P(\bar{x} > 180) = P\left(z > \dfrac{180-173}{5}\right) = P(z > 1.4) = 1 - P(z < 1.4) = 1 - 0.9192 = 0.0808$.

8.44. (a) The sample mean of 100 light bulbs is approximately distributed as $N\left(750, \dfrac{120}{\sqrt{10}}\right) = N(750, 12)$. The consumer agency finds that a sample mean life is 735, which is a value outside the confidence interval $750 \pm 2 \times 12$. Thus the manufacturer's claim is in doubt.

(b) Margin of error.