AMS 102: HOMEWORK 7

SOLUTIONS

Chapter 7

7.47. (a) $P(X = 4) = 1 - (0.1 + 0.1 + 0.2 + 0.4) = 0.2$.
(b) $P(X = 1) = 0.1$.
(c) $P(X < 3) = P(X = 0)+P(X = 1)+P(X = 2) = 0.1+0.1+0.2 = 0.4$.
(d) $P(X = 1|X < 3) = \dfrac{P(X = 1)}{P(X < 3)} = \dfrac{0.1}{0.4} = 0.25$.

7.54. We need to compute expected net return for Option 2. If the coin falls heads on both tosses, the payoff is $\$8 - \$3 = \$5$; otherwise the payoff is $-\$3$ (the entry fee). The coin falls head on both tosses with probability $0.5 \cdot 0.5 = 0.25$. The probability of the other option is $1 - 0.25 = 0.75$. Thus, the expected net return is $0.25 \times \$5 + 0.75 \times (-\$3) = -\$1$. Thus Option 1 is more attractive.

7.60. (a) $P(X = 2) = \binom{10}{2}(0.4)^2(0.6)^8 \approx 0.1209$.
(b) $P(X > 8) = P(X = 9) + P(X = 10) = \binom{10}{9}(0.4)^9(0.6)^1 + (0.4)^10 \approx 0.0016 + 0.0001 = 0.0017$.
(c) $P(X \leq 7) = 1 - P(X > 7) = 1 - (P(X = 8) + P(X = 9) + P(X = 10)) = 1 - \left(\binom{10}{8}(0.4)^8(0.6)^2 + \binom{10}{9}(0.4)^9(0.6)^1 + (0.4)^{10}\right) \approx 1 - (0.0106 + 0.0016 + 0.0001) = 1 - 0.0123 = 0.9877$.
(In principle, we could compute $P(X \leq 7)$ as $P(X = 0) + P(X = 1) + P(X = 2) \cdots + P(X = 7)$ but this would mean computing eight probabilities separately. The method above is faster.)
(d) $P(X = 1.5) = 0$ (X is the number of successes in 10 trials, so must be integer).
(e) $E(X) = np = 10 \cdot 0.4 = 4$.
(f) $Var(X) = npq = 10 \cdot 0.4 \cdot 0.6 = 2.4$.

7.68. (a) $P(x < 10) = P\left(z < \dfrac{10 - 20}{5}\right) = P(z < -2) = 0.0228$.
(b) $P(x > 32) = P\left(z > \dfrac{32 - 20}{5}\right) = P(z > 2.4) = 1 - P(z < 2.4) = 1 - 0.9918 = 0.0082$.

7.90. This is an example of binomial distribution with three trials.

(a) The probability of success in each trial is 98%, i.e. 0.98 (here success is "battery is not defective"). The probability that no batteries out of 3 are defective is $(0.98)^3 \approx 0.94$.

(b) Same as in (a), only now the probability of success in each trial is 80%, i.e. 0.8. The probability is thus $(0.8)^3 = 0.512$.

7.98. Another case of binomial distribution with $n = 10$ (each rabbit is a trial) and the probability of success $p = 0.2$ (success is "carries a gene for long hair").

(a) The expected number is $np = 10 \cdot 0.2 = 2$.

(b) $P(X > 3) = 1-P(X \leq 3) = 1-(P(X = 0)+P(X = 1)+P(X = 2)+P(X = 3)) = 1-\left((0.8)^{10} + \binom{10}{1}(0.2)^1(0.8)^9 + \binom{10}{2}(0.2)^2(0.8)^8 + \binom{10}{3}(0.2)^3(0.8)^7\right) \approx 1 - (0.1074 + 0.2684 + 0.3020 + 0.2013) = 0.1209.$