AMS 102: HOMEWORK 11

SOLUTIONS

Chapter 1

1.20. (b): if the null hypothesis is true, the chance of falsely rejecting it is 0.05. More specifically, we would reject H_0 if the p-value of our data is less than 0.05. So, if H_0 is true there is still the 0.05 probability to get the p-value less than 0.05.

1.21. (a) H_0: The box is Box A; H_1: The box is Box B.

(b) One-sided to the left. Box B contains more low-value tokens than Box A. If we assume that our box is Box A, the values for Box B will be to the left.

(c) Reject H_0 if the selected token is at most \$5. The Type I error is: to conclude that the box is Box B (i.e. reject H_0), when it's actually Box A. If we are to commit this error in no more than 10% of all possible selections, only two tokens ($2/25 < 10\%$) should lead to the choice of Box B. Two lowest tokens have values \$0 and \$5.

(d) $\alpha = 2/25$ (2 out 25 tokens lead to a Type I error)

(e) We will commit a Type II error if we draw from Box B a token with the value higher than \$5. There are 11 tokens like this. Hence, $11/25$.

(f) Reject H_0 and conclude that we've got Box B.

1.31. (a) One-sided to the left.

(b) A Type I error would be to reject H_0 (i.e. roll a 1) when it's true (i.e. when the die is fair). If the die is fair the chance of rolling 1 is $\alpha = 1/6$.

A Type II error would be not to reject H_0 (i.e. roll anything but 1) when it's false (i.e. when the die is biased). If the die is biased the chance of not rolling 1 is $\beta = 2/10 + 2/10 + 1/10 + 1/10 + 1/10 + 1/10 = 7/10$.

(c) The p-value is $P(\text{outcome} \leq 2) = 2/6$. The p-value is greater than α, hence we conclude that the die is Die A.

Chapter 9

9.5. (b). This is the only p-value less than α.

9.7. (a) The population are pregnant women who spend 1 to 20 hours per week with a computer.

(b) The sample proportion for the given sample is $\hat{p} = 155/697 \approx 0.222$.

Under the null hypothesis, the sampling distribution is $N\left(0.2, \sqrt{\frac{(0.2)(1-0.2)}{697}}\right) \approx$

$N(0.2, 0.0151)$. The z-score is $\dfrac{0.222 - 0.2}{0.0151} \approx 1.47$. The p-value is $P(z > 1.47) = 1 - P(z < 1.47) = 1 - 0.9292 = 0.0708$.

(c) Since $0.0708 > 0.01$, the results are not statistically significant at the 1% level and we cannot reject H_0.

9.17. (a) 0.45

(b) Under H_0 the sampling distribution is $N\left(0.2, \sqrt{\dfrac{0.2(1-0.2)}{600}}\right) \approx N(0.2, 0.016)$. The z-score for $\hat{p} = 0.45$ is $\dfrac{0.45 - 0.2}{0.016} = 15.625$. $P(z > 15.625) = 1 - P(z < 15.625) \approx 1 - 1 = 0$.

(c) Since p-value is almost 0, it is less than α. We reject H_0.

(d) The power is 1-β, where β is the probability of committing a Type II error, i.e. failing to reject H_0 when it's false. We will not reject H_0 if p-value is higher than 0.05. The chart implies that the z-score must be below 1.645. This means that $\hat{p} < 0.226$. If H_0 is false, the $\hat{p}$ is distributed as $N\left(0.4, \sqrt{\dfrac{0.4(1-0.4)}{600}}\right) = N(0.4, 0.02)$. $P(\hat{p} < 0.226) = P\left(z < \dfrac{0.226 - 0.4}{0.02}\right) = P(z < -8.7) \approx 0$. Hence $\beta \approx 0$ and the power of the test is approximately 1.