# COMBINATORICS

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*Problem* 1. Can a $2018 \times 2018$ grid be tiled with rotations and reflections of the $L$-tetromino?

*Problem* 2. Each edge of a complete graph on $2k$ vertices is removed with probability $\frac{1}{2}$. Prove that with probability greater than $\frac{1}{4^k}$, the maximum degree of the remaining graph is at most $k-1$.

*Problem* 3. Let $n$ be a fixed positive integer. How man ways are there to write $n$ as a sum of positive integers,

$$n = a_1 + a_2 + \cdots + a_k,$$

with $k$ an arbitrary positive integer and $a_1 \leqslant a_2 \leqslant ... \leqslant a_k \leqslant a_1 + 1$?

*Problem* 4. How many $n$-digit numbers whose digits are in the set $\{2, 3, 7, 9\}$ are divisible by 3?

*Problem* 5. Given a positive integer $n$, what is the largest $k$ such that the numbers $1, 2, ..., n$ can be put into $k$ boxes such that the sum of the numbers in each box is the same?

*Problem* 6. Let $B$ be a set of more than $2^{n+1}/n$ distinct points with coordinates of the form $\{\pm 1, ..., \pm 1\}$ in $n$-dimensional space with $n \geqslant 3$. Show that there are three distinct points in $B$ which are the vertices of an equilateral triangle.

*Problem* 7. A set of $n$ points is given in the plane such that the distance between any two of them is greater than 1. Prove that one can choose at least $\frac{n}{7}$ of these points such that the distance between any two of them is greater than $\sqrt{3}$.

*Problem* 8. Let $n \geqslant 2$ be an integer and $T_n$ be the number of nonempty subsets $S$ of $\{1, 2, 3, ..., n\}$ with the property that the average of the elements in $S$ is an integer. Prove that $T_n - n$ is always even.

*Problem* 9. Prove that it is not possible to color the squares of a $11 \times 11$ grid using three colors such that no four squares whose centers form the vertices of a rectangle with sides parallel to the sides of the grid have the same color.

*Problem* 10. Suppose a finite number integer arithmetic progressions are given, such that every positive integer belongs to exactly one of them. Prove that two of these progressions have the same difference.