# PROBABILITY

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*Problem* 1. Let $v$ and $w$ be distinct, randomly chosen roots of $z^{1997} - 1 = 0$. Find the probability that $\sqrt{2+\sqrt{3}} \leqslant |v + w|$.

*Problem* 2. A bag contains 1993 red balls and 1993 black balls. We remove 2 balls from the bag repeatedly and discard both if they are the same color, or discard the black ball but not the red ball if they are different colors. What is the probability that the process terminates with one red ball in the bag?

*Problem* 3. What is the probability that a randomly chosen permutation of $\{1, 2, ..., n\}$, $n \geqslant 2$ has 1 and 2 in the same cycle.

*Problem* 4. Given the independent events $A_1, ..., A_n$ with probabilities $p_1, ..., p_n$, find the probability that an odd number of the events occur.

*Problem* 5. A coin is tossed $n$ times. What is the probability that a pair of consecutive heads appears in the sequence?

*Problem* 6. An unbiased coin is tossed $n$ times. Find a formula, in closed form, for the expected value of $|H - T|$, where $H$ is the number of heads and $T$ is the number of tails.

*Problem* 7. Prove the identities $\sum_{k=1}^{n} \frac{1}{(k-1)!} \sum_{i=0}^{n-k} \frac{(-1)^i}{i!} = 1$ and $\sum_{k=1}^{n} \frac{k}{(k-1)!} \sum_{i=0}^{n-k} \frac{(-1)^i}{i!} = 2$.

*Problem* 8. Let $0 < \alpha < \beta$. If two points are selected at random from a segment of side length $\beta$, what is the probability that the distance between them is less than $\alpha$?

*Problem* 9. What is the probability that three points selected on a circle lie on a semi-circle?

*Problem* 10. Let $n \geqslant 4$ and suppose points $P_1, ..., P_n$ are randomly distributed on a circle. What is the probability that the convex $n$-gon formed by the points has an acute angle?