PROBABILITY

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Problem 1. Let v and w be distinct, randomly chosen roots of the equation $z^{1997} - 1 = 0$. Find the probability that $\sqrt{2+\sqrt{3}} \leqslant |v + w|$.

Problem 2. A bag contains 1993 red balls and 1993 black balls. We remove two balls at a time repeatedly and

(1) discard them if they are of the same color
(2) discard the black ball and return the red ball to the container if different colors.

What is the probability that the process terminates with one red ball in the bag.

Problem 3. Find the probability that in a group on n people there are two with the same birthday ignoring leap years.

Problem 4. What is the probability that a permutation of the first n positive integers has the numbers 1 and 2 within the same cycle.

Problem 5. Prove the identities

$$\sum_{k=1}^{n} \frac{1}{(k-1)!} \sum_{i=0}^{n-k} \frac{(-1)^i}{i!} = 1,$$

$$\sum_{k=1}^{n} \frac{k}{(k-1)!} \sum_{i=0}^{n-k} \frac{(-1)^i}{i!} = 2.$$

Problem 6. An exam consists of 3 problems selected randomly from a list of $2n$ problems, where n is an integer greater than 1. For a student to pass, he needs to solve correctly at least two of the three problems. Knowing that a certain student knows how to solve exactly half of the $2n$ problems, find the probability that the student will pass the exam.

Problem 7. Find the probability that in the process of repeatedly flipping a coin, one will encounter a run of 5 heads before one encounters a run of 2 tails.

Problem 8. Given the independent events $A_1, A_2, ..., A_n$ with probabilities $p_1, p_2, ..., p_n$, find the probability that an even number of these events occurs.

Problem 9. What is the probability that the sum of two randomly chosen numbers in the interval $[0, 1]$ does not exceed 1 and their product does not exceed $\frac{2}{9}$?

Problem 10. Let α and β be given positive real numbers, with $\alpha < \beta$. If two points are selected at random from a straight line segment of length β, what is the probability that the distance between them is at least α?

Problem 11. Let $n \geqslant 4$ be given, and suppose that the points $P_1, P_2, ..., P_n$ are randomly chosen on a circle. Consider the convex n-gon whose vertices are these points. What is the probability that at least one of the vertex angles of this polygon is acute?

Problem 12. At each plane lattice point a positive number is placed such that each is the average of its four neighbors. Show that all of the numbers are the same.