MATH 533, SPRING 2022, HW6

DUE MARCH 7

Problem 1. Let μ, ν be Radon measures on X, Y, not necessarily σ-finite. If f is a nonnegative l.s.c. function on $X \times Y$, show that $x \to \int f_x d\nu$ and $y \to \int f^y d\mu$ are Borel measurable and $\int f d(\mu \hat{\times} \nu) = \iint f d\mu d\nu = \iint f d\nu d\mu$.

Problem 2. The following gives an example of a smooth function not equal to its Taylor expansion at 0. Let $f(t) = e^{-1/t}$ for $t > 0$, $f(t) = 0$ for $t \leq 0$. Check that

(1) For $k \in \mathbb{N}$ and $t > 0$, $f^{(k)}(t) = P_k(1/t)e^{-1/t}$ where P_k is a polynomial.
(2) $f^{(k)}(0)$ exists and is equal to 0 for all k.

Problem 3. If $f \in L^\infty$ and $\|f^y - f\|_\infty \to 0$ as $y \to 0$, then f agrees a.e. with a uniformly continuous function.