MATH 342, SPRING 2025 PRACTICE FINAL

Each problem is worth 10 points.

Problem 1.

a. Find $\frac{3+5i}{2+i}$.

b. Find all values of $(1+i)^{\sqrt{2}}$.

c. Solve $\cos z = 6i$.

Problem 2. Find the orders of zeros and poles and calculate all residues of the function $\frac{\sin \pi z}{z^4+1}$.

Problem 3. Find the order z^4 part of the MacLaurin expansion of the function $\frac{e^{z^2}}{z^2+4}$.

Problem 4. Calculate the positively oriented contour integral $\frac{1}{2\pi i}\int_{|z|=2}\frac{\sin(\cos z)}{(z^2-1)(z+3)}dz$.

Problem 5. Find the Laurent expansion about 0 of $f(z) = \frac{1}{(z^2+1)(z+2)}$ in $|z| < 1$ and in $|z| > 2$.

Problem 6. Evaluate the integral $\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)(x^2+4)}$.

Problem 7.

a. How many zeros of the function $z^9 + 7z^4 + 2z + 3$ are enclosed in the contour $|z| = 1$?

b. What value of the function $f(z) = \frac{e^z(\sin z)^2}{z^2}$ at $z = 0$ would make the function entire?