MATH 320, FALL 2022 PRACTICE MIDTERM 2

OCTOBER 27

Each problem is worth 10 points.

Problem 1. Give the proof from lecture that a continuous function on a closed bounded interval attains its maximum and minimum.

Problem 2. Suppose f and g are n times differentiable at a point a. Prove Leibniz's formula

$$(fg)^{(n)}(a) = \sum_{k=0}^{n} \binom{n}{k} f^{(k)}(a) g^{(n-k)}(a).$$

Problem 3. Let $f : (a, b) \to \mathbb{R}$ be differentiable and let f be twice differentiable at a point $x \in (a, b)$. Prove

$$f''(x) = \lim_{h \to 0} \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}.$$

Give an example of a function f for which the limit exists but the function does not have a second derivative at the point.

Problem 4. A function $f : \mathbb{R} \to \mathbb{R}$ is said to be *periodic* on $\mathbb{R}$ if there exists a number $p > 0$ such that $f(x + p) = f(x)$ for all $x \in \mathbb{R}$. Prove that a continuous periodic function on $\mathbb{R}$ is bounded and uniformly continuous on $\mathbb{R}$.