MATH 319/320, SPRING 2020 MIDTERM 1

FEBRUARY 27

Each problem is worth 10 points.

Problem 1. Let $F_0 = F_1 = 1$, $F_{n+1} = F_n + F_{n-1}$ be the Fibonacci sequence. Prove that for all natural numbers n, $F_n \leq 2^n$.

Problem 2. Prove that $x^2 = 3$ does not have a rational solution, but that it has a positive real solution.

Problem 3. Prove that each non-empty interval of $\mathbb{R}$ contains both rational and irrational numbers.

Problem 4. Show that if $z_n = (a^n + b^n)^{\frac{1}{n}}$ where $0 < a < b$, then $\lim(z_n) = b$.

Problem 5. Let $S \subset \mathbb{R}$ be a bounded set and $S_0 \subset S$ a non-empty subset. Prove

$$\inf S \leq \inf S_0 \leq \sup S_0 \leq \sup S.$$